

LOGIC AS A PUZZLE-SOLVING ACTIVITY

La lógica como una actividad de resolución de enigmas

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Abstract

Some authors have recently argued in favor of anti-exceptionalism about logic. The general idea is that logic is not different from the other sciences, and its principles are as revisable as scientific principles. This paper has three sections. In section 1, I discuss the meaning of anti-exceptionalism and its place in contemporary logic. In section 2, I analyze some recent developments on this topic by Williamson (2017) and Hjortland (2017), which will motivate my view. In section 3, I propose a puzzle-solving perspective on logical practice. According to my view, there is a common methodology, in which scientists may use non-classical in order to solve some specific puzzles, but classical logic stays in a privileged position, as a common language and as a general theory of reasoning. This role cannot be fulfilled by other logics, and therefore the comparison between classical and non-classical logic is not like a regular comparison between competing hypotheses in science. The methodology of logical practice is therefore not abductive, at least in many important cases. Classical logic is not the “best available theory”, but the fundamental piece of our scientific methodology. My position is still anti-exceptionalist: logic is like any other science, or at least like any other science which can be characterized by a puzzle-solving methodology.

Key words: Anti-Exceptionalism; Logical Revision; Scientific Methodology; Puzzle-Solving; Non-Classical Logics.

Resumen

Algunos autores han defendido recientemente el anti-excepcionalismo sobre la lógica. La idea general es que la lógica no es distinta de las otras ciencias y sus principios son tan revisables como los principios científicos. Este artículo tiene tres secciones. En la sección 1, discuto el significado del anti-excepcionalismo y su lugar en la lógica contemporánea. En la sección 2, analizo algunos desarrollos recientes sobre el tema hechos por Williamson (2017) y Hjortland (2017), que motivarán mi enfoque. En la sección 3, propongo una perspectiva de resolución de enigmas sobre la práctica lógica. De acuerdo con mi posición, hay una metodología común, en la cual los científicos pueden usar lógicas no clásicas para resolver algunos enigmas científicos, pero la

lógica clásica permanece en una posición privilegiada, como lenguaje común y teoría general del razonamiento. Este rol no lo pueden cumplir otras lógicas y por eso la comparación entre lógica clásica y lógicas no clásicas no es como una comparación usual entre hipótesis rivales en ciencia. Por lo tanto, la metodología de la práctica lógica no es abductiva, al menos en varios casos importantes. La lógica clásica no es la “mejor teoría disponible”, sino la pieza fundamental de nuestra metodología científica. Mi posición sigue siendo anti-excepcionalista: la lógica se parece a las otras ciencias, o al menos a aquellas que se pueden caracterizar con una metodología de resolución de enigmas.

Palabras clave: Anti-excepcionalismo; Revisión lógica; Metodología científica; Resolución de enigmas; Lógicas no clásicas.

1. Introduction

In recent years, many authors have been discussing the so-called “anti-exceptionalism about logic”. According to anti-exceptionalism, logic is not necessarily different from natural science. Logical principles can be revised and questioned in the same way as scientific hypotheses. This is how Hjortland describes the idea (2017, p. 2):

Logic isn't special. Its theories are continuous with science; its method continuous with scientific method. Logic isn't a priori, nor are its truths analytic truths. Logical theories are revisable, and if they are revised, they are revised on the same grounds as scientific theories.

In other words, anti-exceptionalism claims that logic is: (i) characterized by a scientific method; (ii) not a priori; (iii) not analytical; (iv) revisable; and (v) revised on the same grounds as scientific theories (this could be interpreted as “empirical”). It is not clear whether the position needs all the properties to apply or just a relevant subset of them.

Anti-exceptionalism is supposedly contraposed to more foundationalist views of logic. According to these views, logic provides some rules which cannot be revised. Logical rules, a foundationalist can say, come from the meaning of logical connectives, so they cannot be revised using empirical evidence or appealing to semantical paradoxes. Alternatively, she can say that the principles are grounded on the rigid structure of the world or the mind, at the point in which illogical statements cannot be represented or understood. From this exceptionalist approach, a principle such as the excluded middle has

the same status as an arithmetical platitude like “ $2+2=4$ ”. Empirical statements have a dynamic nature, but those basic logical principles are simply beyond discussion.

One of the rhetorical problems of anti-exceptionalism, however, is that it is not easy to find “exceptionalist” logicians. A large portion of the contemporary discussion on paradoxes and truth presupposes that adopting non-classical logics is, at least, an open option. The strict version of logical exceptionalism can certainly be found in early analytic philosophers, such as Wittgenstein (“There can never be surprises in logic”, *TLP* 6.12.41), but it is not mainstream today.

Some recent papers focus on methodological issues. They take anti-exceptionalism as the adoption of a so-called “abductive” method of logical revision (Priest, 2014; Williamson, 2017; Hjortland, 2017). According to abductivism, “we choose the theory which best meets those criteria which determine a good theory” (Priest, 2014, p. 217). The set of criteria includes fit do data, but it may also include other epistemic virtues such as simplicity, ad-hocness, unifying power, fruitfulness, etc. Abductivism presupposes a comparison between different theories: “We come to be justified in believing a certain logical theory L because it better accommodates the relevant data, and possesses the relevant theoretical virtues to a greater extent, than competing theories” (Martin & Hjortland, 2020, p. 2). This abductive version of anti-exceptionalism is opposed to other two epistemologies of logic: *rationalism* (Bealer, 1998)¹, which holds that we learn logic by rational insight, and *semanticism* (Boghossian, 2000), according to which we learn logic by understanding the meaning of logical expressions.²

In a recent paper, Martin and Hjortland (2020) identify anti-exceptionalism with the more general idea that logic has a scientific methodology: *Methodological Anti-Exceptionalism* means that “Theory choice within logic is similar in important respects to that of the recognized sciences” (p. 2). Logical abductivism is one version of this idea (p. 2, fn. 1). However, methodological anti-exceptionalism should not be identified with abductivism, for logic might work with a scientific methodology which is not abductive. Martin and Hjortland,

¹ This version of Rationalism is not only an epistemology of logic but a general meta-philosophy. The idea is that we learn about philosophical concepts by rational intuition.

² Boghossian’s semanticist approach is not regarded as a method to find the correct logic, but rather as a way of justifying our most basic logical beliefs. Williamson (2003) observes the incompatibility between this approach and the general discussion about the validity of some logical inferences.

for example, argue that the methodology of logic is based on prediction and explanation. According to this view, logic is supposed to “predict and explain” validity sentences.³ The general idea is that we construct a theory (using definitions, rules, and laws) to explain the validity of some intuitively valid informal arguments, and then we predict the validity of other arguments. We need to check whether the new predictions hold; if not, the theory (or some auxiliary hypotheses) must be revised. On the other hand, Payette and Wyatt (2018) claim that logic works with the methodology of functional explanation.⁴ Under this approach, logicians want to explain the validity or invalidity of some inferences that involve certain structure and vocabulary; the aim is not to find a general logic for the entire natural language. Therefore, every logical theory will have some degree of inaccuracy, as it has a limited scope, but this is not necessarily problematic.

In this paper, I will support Methodological Anti-Exceptionalism in general, and, like these authors, I will also reject the general picture of logical abductivism. The abductive methodology, I will argue, ignores some central aspects of the logical practice. I will claim that, in general, logical inquiry can be characterized by a puzzle-solving methodology.

In section 2, I discuss some recent proposals which belong to the abductivist version of anti-exceptionalism: Williamson (2017) and Hjortland (2017). Williamson argues that an abductive method of logical revision will decide in favor of classical logic, which is stronger and more fundamental for science than any other logical theory. According to Hjortland, and *contra* Williamson, abductive anti-exceptionalism does not necessarily result in classical logic. I argue that both approaches have shortcomings, which will hopefully be solved by my approach.

In section 3, I develop a puzzle-solving version of logical anti-exceptionalism.⁵ I claim that many contemporary developments in non-classical logic have a limited scope: they aim to solve certain puzzles

³ Two important clarifications should be made about this (Martin & Hjortland, 2020, p. 4). First, they do not believe that predictivism is the *only* methodology of logic (although they do not elaborate more on that). Secondly, this view of prediction is obviously non-temporal. The predicted cases are just cases that were not explicitly considered when the theory was built.

⁴ The methodology of functional explanation that Payette and Wyatt use was described by Andrea Woody (2015). However, a detailed analysis of her view is out of the scope of this paper.

⁵ As it will be clear later, I do not claim that logic *always* works with this puzzle-solving dynamic. There is a degree of methodological pluralism in logic, as there is also in science.

using some specific rules and methods. The abductive model of logical revision is therefore not adequate, for the reason why classical logic is stronger in many evaluations, and has a privileged epistemic status, is that the common language of our puzzle-solving methodology is classical. Classical logic is not just a theory among others, but a set of principles and methods which cannot be abandoned using specific pieces of evidence such as semantic paradoxes.

However, I will also claim that non-classical logics have an important and legitimate role in logical practice. These approaches provide new methods, tools and perspectives to solve some interesting theoretical puzzles. They certainly contribute to scientific progress in logic. Still, they generally do not represent a genuine challenge for classical logic in its fundamental role.

2. Classical and Non-Classical Abductivism

In this section, I will analyze two central contributions in the literature about anti-exceptionalism: Williamson (2017) and Hjortland (2017). I will explain their positions and discuss some objections that will lead us to a more comprehensive view. I will also explain in which sense my own view (developed in section 3) will be different from both.

One of the main proponents of anti-exceptionalism today is Timothy Williamson (2017). He claims that logic is not special, for it just involves unrestricted generalizations of true sentences. For example, the tautology $\varphi \vee \neg\varphi$ expresses the generalization $\forall p (p \vee \neg p)$.⁶ It is neither obvious nor analytic that the statement has no exceptions; it is simply true (cf. p. 328). There is not a big difference, apart from generality, between testing logical statements and geological statements. And the reasons for revising logic, Williamson says, are to be found in the exceptions to these general statements.

This does not mean that given any apparent exception, the logical principles will be abandoned. On the contrary, Williamson says that we should follow the abductive method; this means, we may revise classical logic only when there is another logic which can improve upon it in different epistemic virtues. If there is no such alternative logic, the tentative counterexamples to logical principles can be explained away by expressive limitations and other resources.

⁶More precisely, Williamson (p. 329) introduces a Universal Generalization function UG , which can be applied to any first-order sentence to obtain a generalization (in first or second order). For example, $UG(a=a) = \forall x(x=x)$. For sentences with predicates, UG gives a second order generalization: $UG(Fa \vee \neg Fa) = \forall F \forall x (Fx \vee \neg Fx)$

Williamson argues that classical logic will be chosen by the abductive methodology. Classical logic has two main advantages, which are *strength* and (what I call) *ubiquity*:

- *Strength* (pp. 336-337). Classical logic is typically stronger than the most used non-classical logics such as Intuitionistic Logic, Relevant Logic, *LP* or *K3*. Classical logic has the deduction theorem, Modus Ponens, Law of Excluded Middle, structural properties, DeMorgan rules, etc. Indeed, it is widely known that one cannot add principles to classical logic (*i.e.* go supraclassical) and maintain the uniform substitution property.
- *Ubiquity*. A bit more controversial, but also widely accepted, is the claim that classical logic is ubiquitous. As Williamson describes it:

[classical logic] has been tested far more severely than any other logic in the history of science, most notably in the history of mathematics, and has withstood the tests remarkably well (p. 338)

Given these two main virtues of classical logic, classicality will prevail in the abductive comparison with other logical theories. Williamson, nevertheless, does not think that this attitude is conservative. Classical logic will not prevail just because we are used to it, but because of its intrinsic features (p. 338).

Williamson admits that classical logic has a shortcoming: it is not completely “fit” to evidence, because of semantic paradoxes. Classical logic cannot keep the unrestricted T-schema, which expresses the meaning of the vernacular concept of truth. Many non-classical logicians take the Liar paradox as the main reason for abandoning classical logic. This is one clear instance of Williamson’s generalizations which cannot satisfy classical laws. Williamson responds to this point that logic is not necessarily concerned with analyzing truth, so this concept has a secondary importance (p. 339).⁷ According to him, it is better to abandon the unrestricted T-Schema and keep the classical rules and principles. Therefore, classical logic remains the best logical theory. Even though classical logic is not perfect (for it cannot contain the unrestricted T-schema), it is still the best logical theory, for it is strong and ubiquitous.

⁷ Another reason Williamson mentions (p. 340) is that truth principles do not have a special importance in science, unlike logical principles which are ubiquitous and necessary for scientific reasoning.

At this point it is hard to see how anti-exceptionalist Williamson's position is. If the virtues under consideration are strength and ubiquity, then classical logic will in principle not be revised in the face of new evidence. Semantic paradoxes, for example, will be incapable of providing a reason for revising classical logic. And non-classical proposals will hardly be considered in this dispute, for they are usually weaker and clearly not so ubiquitous as classical logic. Williamson (p. 340) quotes Quine, who claimed that solving paradoxes in non-classical logic involved to "lay more fields to waste". His methodology is therefore only apparently abductive; the chosen epistemic virtues will make classical logic win against any other possible logic, so there is not a genuine competition between logical theories to begin with.

In the next section I will provide a theory about the methodology of logic where Williamson's classical intuitions are captured: classical logic is indeed a privileged theory, with a broad application in scientific research that cannot be ignored. However, this will not imply that non-classical developments are doomed to failure. In order to achieve this, we need to reject the abductive picture, where non-classical logics are just "rivals" of classical logic. Many non-classical logics are not going to succeed if the idea is replacing classical logic; but I will argue that this is not the aim of these theories in general.

Hjortland (2017) develops another version of abductivism, with a rather non-classical spirit. He responds to Williamson that, even though anti-exceptionalism is right, and the methodology of logic is abductive, it does not necessarily select classical logic. Hjortland does not defend a specific logic, but just the idea of going non-classical, or even substructural (p. 2):

(...) abductivism about logic does not lead to classical logic. It does not follow, however, that abductivism supports a specific nonclassical logic.

Hjortland claims that the fact that most scientists use classical logic is not a conclusive argument in favor of it. He argues that mathematical proofs are not grounded in classical logic but in classical *reasoning*. Mathematics does not need unrestricted logical principles, and it would work equally fine with classical logic restricted to mathematical reasoning:

Mathematical proofs do contain an abundance of instances of classical principles: applications of classical *reductio ad absurdum*,

conditional proof, disjunctive syllogism, the law of absorption, etc. The emphasis, however, should be on the fact that these are *instances* of classical principles. The mathematical proofs do not rely on any of these principles being unrestricted generalizations of the form that Williamson defends (pp. 22-23)

If this is right, the ubiquity of classical logic in different sciences is not necessarily a reason for regarding it as the correct logic. Recapture theorems can explain why in some specific domains, such as mathematics, classical logic can work, while the general logic for the whole domain of objects is subclassical.⁸ Classical logic is necessary for many sciences; but Recapture arguments can be used to show that even in that case, classical logic is not necessarily the correct logic, but the most useful one in specific domains.

It is worth remarking, however, that Hjortland's paper does not advocate for a specific non-classical logic. Hjortland argues, instead, that abductivism makes room for a kind of language-relative logical pluralism, where (*e.g.*) the logic is classical for the language without the truth predicate, but it is non-classical for the language with the truth predicate (p. 23). However, this does not solve the central question: *which* non-classical logic does characterize the truth predicate? It is not enough to take for granted that there is one, without arguing in favor of one. As I will claim below, there is an important reason why we can agree about the logic for the unproblematic part of the language, but we cannot agree about the logic for the problematic part. This is essential for contemporary logical research.

In this paper, I will follow Hjortland in making room for non-classical logic. However, I will not claim that non-classical logics are in an "abductive" competition with classical logic. What is more, I will not assume that there is one "best" non-classical logic that can work as a logic for the whole language, or even for the truth predicate. The idea that there should be a general "theory of validity" that works in every case, and should be chosen by abductive criteria, is not represented by actual practice.

I will argue against abductivism in general: logical practice is not a dispute between logics to be crowned as "the correct logic". There is

⁸ A Recapture result for a non-classical logic *NCL* shows that if Γ implies ϕ in classical logic, $\Gamma \cup \Delta$ implies ϕ in *NCL*. For example, in the case of propositional *K3*, Δ can include $p \vee \neg p$ for every propositional letter p in the premises or the conclusion (Beall, 2013b). There is certain amount of discussion regarding other concepts of Recapture, which can apply to paraconsistent logics (in particular to systems such as *LP* where you cannot express the fact that a proposition is necessarily consistent).

possibly no fact of the matter which can determine which one is the *best* theory of truth, or the best logic in general. However, this does not mean that developing alternative theories is a waste of time. Logicians are not fighting for a crown; in many cases they are exploring new territory, which is also a valuable task. Once we abandon the abductive picture where non-classical logics are “competing” against classical logic, we can understand better the logical practice, its dynamics and its value.

3. The Puzzle-Solving Methodology of Logic

3.1. *Two motivations for non-classical logic*

The approach in this paper will be practice-based (cf. Martin, 2020): the main aim of my model is to explain and illuminate the practice of actual logicians. Admittedly, every model has a normative aspect, so I do not pretend to provide a purely descriptive approach. But I will try to understand the specific ways in which philosophers are typically involved in strategies of so-called logical revision, and my position will be grounded on that “sociological” evidence.

As I will argue, there are two different activities of philosophical and logical practice which normally motivate non-classical perspectives: changing the general foundations of logic and solving specific puzzles. This distinction is sometimes a matter of degrees, but I hope it becomes clear with some examples.

Advocates of logical revision used to proceed in a more foundational –and revolutionary– way. For example, intuitionism involves a constructivist point of view regarding mathematics and reality, and it is not supposed to solve a specific problem, but to put logic and mathematics on more solid grounds. According to Brouwer (1908, p. 107): “logical deductions which are made independently of perception, being mathematical transformations in the mathematical system, may lead from scientifically accepted premises to an inadmissible conclusion”. Dummett (1978, p. 215) explains this revolutionary approach by distinguishing it from a pluralistic (“eclectic”) point of view:

I am, thus, not concerned with justifications of intuitionistic mathematics from an eclectic point of view, that is, from one which would admit intuitionistic mathematics as a legitimate and interesting form of mathematics alongside classical mathematics: I am concerned only with the standpoint of the intuitionists themselves, namely that classical mathematics employs forms of

reasoning which are not valid on any legitimate way of construing mathematical statements.

The same can be said about relevant logics in the beginnings. Relevant logicians (in particular, Anderson & Belnap, 1962) criticized the classical notion of logical consequence and developed an alternative conception which could respond to their worries. According to Anderson and Belnap, classical logic failed at capturing the ordinary use of “entailment” and “implication” in natural language and mathematics. They claim that consequence needs more than truth preservation, for the conclusion must be deducible in a relevant way from the premises (p. 31):

Of course, we *can* say “Assume that snow is puce. Seven is a prime number.” But if we say “Assume that snow is puce. *It follows that* (or *consequently*, or *therefore*, or *it may validly be inferred that*) seven is a prime number”, then we have simply spoken falsely.

In any case, relevant logic did not appear as a puzzle-solving logic, but mostly as a revolutionary point of view regarding inference.⁹ Norman and Sylvan (1989, p. 10) are clear about this enterprise:

Relevantism rejects classical logic as incorrect, and adopts instead a relevant logic as supplying the basis of a theory of correct argument. In significant respects relevantism is like intuitionism; it is likewise anti-classical, but bases its program on relevant rather than intuitionist logic. Like intuitionism, relevantism sets a substantial theoretical program: that of reworking logic and what hinges materially upon it, such as the foundations of mathematics and science.

However, this revolutionary aim of changing the foundations of logic is not shared by most actual researchers. As we will see, contemporary developments advocating for non-classical logics can usually be identified with a puzzle-solving activity. They normally use non-classical logic to solve semantic paradoxes (Liar, Curry, V-Curry, etc.), or metaphysical puzzles (Sorites, etc.). However, this non-classicality has some limits; as it was observed many times, these non-classical theories often use

⁹ Admittedly, some relevant logicians mentioned a series of puzzles (for example, the Positive Paradox), but the main aim of relevant logic was not to solve these puzzles but to use them as symptoms of deeper concerns regarding logical consequence.

a classical metatheory, and they are not supposed to transform the structure of ordinary mathematical reasoning.¹⁰

A standard piece of research in this area might offer a non-classical solution (paraconsistent, paracomplete, fuzzy, substructural, etc.) to a certain puzzle, and then show *using classical logic* that this non-classical logic does not lead to new paradoxes and is approximately conservative with respect to classical principles. These solutions are sometimes affected by new puzzles, such as revenge paradoxes; and subsequent research may address these revenge problems specifically.

3.2. *The existence of a puzzle-solving methodology*

If the analysis above is correct, then the comparison between classical and non-classical logic is not like the comparison between competing theories in science, at least in these puzzle-solving scenarios. The abductive method is simply not fit for this task. In many cases, we are not seeking to truly adopt a non-classical logic as a general theory of reasoning. On the contrary, there is a *puzzle-solving methodology* at work.¹¹ Inside this methodology, non-classical solutions to paradoxes are encouraged and accepted as theoretical contributions, but classical logic cannot be replaced as a general theory of reasoning¹² and as a common language.

The solutions to puzzles typically resemble each other, for they are usually based on previous results. I will borrow the notion of “exemplar” from Kuhn (1970): exemplars are the concrete solutions to problems that researchers take as models for their activity. Some exemplars for logic are the usual proofs you can find in an intermediate logic textbook, such as a Henkin completeness proof, or a Cut-elimination result; in the specific area of non-classical theories of truth, the obvious exemplar is Kripke’s fixed-point theorem. Most logical research, either on classical

¹⁰ It is worth-remarking that Williamson’s (2017) paper on abductive methodology is focused on this puzzle-solving kind of logical revisionism, as the title suggests: “Semantical paradoxes and abductive methodology”.

¹¹ This idea is obviously inspired on Kuhn’s (1970) famous theory of scientific paradigms. However, as an anonymous referee observed, Kuhn’s theory is very sophisticated (including, for example, claims about incommensurability) and I would not claim that it applies entirely to logical research. Therefore, I cannot say that logic has a Kuhnian methodology. At best, it has a methodology which resembles Kuhn’s theory in many aspects.

¹² Here, “reasoning” refers to a specific activity: showing formally how some premises imply a conclusion. I am not talking about the psychology of reasoning, which is in many ways disconnected from logic.

or non-classical logic, uses some of the techniques of exemplars. And these exemplars are always formulated in classical logic. This is one of the reasons why classical logic is in most cases not optional.

The puzzle-solving methodology is repeated in many pieces of non-classical logic. The methodology permits the use of non-classical systems in order to solve paradoxes or logical puzzles, but the metalanguage must stay classical. The range of accepted solutions are classical or non-classical theories of semantic and metaphysical concepts which can be proved to be consistent, complete or non-trivial. And these proofs are based on the exemplars. The methodology cannot always determine a winner, but it provides some rules for the admissibility of different solutions.

Now we can explore some examples. Philosophers like Kripke (1975), Beall (2009), Field (2008), or Ripley (2012) can illustrate the puzzle-solving activity of many contemporary logicians. They developed theories which could express the transparent notion of truth and proved the main meta-theoretical results in classical logic. In addition, some of them provided Recapture results, which might recover classical reasoning for mathematics inside the non-classical theory for truth.

Kripke's *Outline of a theory of truth* (1975), maybe the most important paper on non-classical solutions to paradoxes, provided a Kleene-based theory of truth, and a philosophical explanation of indeterminacy based on un-groundedness. In Kripke's theory, sentences φ and $T(\varphi)$ are intersubstitutable. This condition (later known as "Transparency" or "Intersubstitutivity") remained as a common desideratum for theories of truth. This paper also includes a fixed-point theorem, which establishes the non-triviality of the truth theory; after Kripke's work, fixed-point theorems became an essential resource for non-classical theories of truth.

Kripke was also skeptical about "logical revision" in general. His theory involved a three-valued approach about pathological sentences, but he did not believe that this implied in any sense a "change of logic". Kripke was not even committed to accepting a third value apart from "truth" and "falsity", and he did not regard the use of classical logic in the meta-language as a problem (pp. 700-701):

I have been amazed to hear my use of the Kleene valuation compared occasionally to the proposals of those who favor abandoning standard logic "for quantum mechanics", or positing extra truth values beyond truth and falsity, etc. (...) conventions for handling sentences that do not express propositions are not in any philosophically significant

sense “changes in logic”. The term “three-valued logic”, occasionally used here, should not mislead. All our considerations can be formalized in a classical metalanguage.

This approach, with Transparency, fixed-point theorems, and classical meta-theory, paved the way for the posterior research. Kripke himself is very clear about the aims of his proposal; he is not providing the “right” theory of truth, but rather a set of conceptual and technical tools to work on this issue (p. 700):

I do not regard any proposal, including the one to be advanced here, as definitive in the sense that it gives the interpretation of the ordinary use of “true”, or the solution to the semantic paradoxes. On the contrary, I have not at the moment thought through a careful philosophical justification of the proposal, nor am I sure of the exact areas and limitations of its applicability. I do hope that the model given here has two virtues: first, that it provides an area rich in formal structure and mathematical properties; second, that to a reasonable extent these properties capture important intuitions. The model, then, is to be tested by its technical fertility.

The history of logic showed that this new area was indeed really “rich in formal structure and mathematical properties”, and that the “technical fertility” of Kripke’s approach was remarkable. Most of the research on non-classical theories of truth was based in one way or another on Kripke’s work. I will mention some examples.

Field (2008) provided a theory of truth based on paracomplete logic, but including also suitable conditionals which satisfy the law of identity (*i.e.* $\varphi \rightarrow \varphi$). These conditionals represent the big difference with Kripke’s approach. His theory also satisfies full Intersubstitutivity, *i.e.* φ and $T(\ulcorner \varphi \urcorner)$ are interchangeable. Field’s central result (2008, ch. 16) is also a fixed-point theorem developed in classical logic. Like Kripke, Field (p. 15) is explicit about preserving classical mathematics:

(...) we ought to seriously consider restricting classical logic to deal with all these paradoxes. In particular, we should seriously consider restricting the law of excluded middle (though not in the way intuitionists propose). I say “restricting” rather than “abandoning”, because there is a wide range of circumstances in which classical logic works fine. Indeed, I take excluded middle to be clearly suspect only for certain sentences that have a kind of “inherent circularity”

because they contain predicates like “true”; and most sentences with those predicates can be argued to satisfy excluded middle too.

In a similar vein, Beall (2009) argues that paradoxes are “spandrels”, *i.e.* unintended consequences which are caused by a transparent theory of truth. According to him, fragments of the language which do not involve semantic concepts can be understood with classical logic. He develops a paraconsistent and transparent theory of truth, with a suitable conditional.¹³ For the fragment without conditionals, Beall’s theory is like Kripke’s, but making Kleene’s third value designated (so the theory becomes dialetheist). As in Field’s approach, Beall’s central result (2009, ch. 2) is also a fixed-point proof developed in classical logic. Beall is also clearly conservative about classical mathematics (p. 112):

(...) we needn’t —and I don’t— see arithmetic as anything more than classical. What is important to remember is that, on my account — as on other standard accounts of *ttruth* [Transparent truth]— we may enjoy a perfectly classical proper fragment of the language. (...) nothing in my account rules out endorsing classical theories, where such theories are written in some suitably proper fragment of the language.

Moreover, in more recent papers, Beall (2013a) explored different methods for recapturing classical validity in paraconsistent settings, such as using multiple conclusions.

Finally, Ripley (2012) provided a theory of truth where the structural rule of Cut fails. The semantics is three-valued: it has a Strong-Kleene matrix, and pathological sentences such as the Liar have value $\frac{1}{2}$. The models of the theory are based on Kripke’s approach. But Ripley modifies the definition of validity: in his theory *ST*, an argument is valid whenever if the premises have value 1, the conclusion has value 1 or $\frac{1}{2}$ (p. 356). Unlike other non-classical approaches, *ST* is conservative with respect to classical logic (p. 359, Corollary 3.7). However, Ripley’s theory does not preserve the structural rule of Cut. In particular, where λ is the liar sentence, $\lambda \models p$ and $\models \lambda$, but $\not\models p$.

There are many other contemporary approaches about paradoxes: three-valued logics with Weak-Kleene matrixes (Gupta & Martin,

¹³ The transparency of truth is the main technical difference between Priest’s (2006) and Beall’s (2009) approach. According to Priest, truth is not transparent, for $\neg\phi$ does not imply $\neg T(\ulcorner \phi \urcorner)$.

1984), four-valued logics (Beall, 2019; Da Re, Pailos & Szmuc, 2018), fuzzy logics (Hájek, Paris & Shepherdson, 2000), five-valued logics including “pathologically true” and “pathologically false” (Beall, 2014), non-contractive logics (Zardini, 2011), non-reflexive logics (French, 2016), etc.

However, among this impressive variety, there is a common pattern. These philosophical logicians solve specific problems such as the Liar paradox, explain their non-classical solutions using classical logic in the meta-language (typically with a fixed-point theorem), show why classical logic can be maintained in non-semantic reasoning, and provide additional responses to revenge problems. Therefore, the contemporary logical development does not look like a battle between rival hypotheses, but more like the exploration of an unknown land, where discovering new points of view has an intrinsic value, provided the agents follow some common rules.

3.3. *Admissibility and progress*

The theory of logical practice I have presented is certainly very open to non-classical contributions. However, I don’t want to imply that every non-classical solution to logical puzzles which satisfies the usual requirements of acceptability (non-triviality, etc.) is equally fine. If my view is correct, this can and will be decided by considering different epistemic values. Some solutions might be internally incoherent, or they might not provide anything interesting or new (from a theoretical or technical point of view).

Still, non-classical logicians are often very pluralistic about the different solutions to the paradoxes. Some contributions do not presuppose that the rival theories are *wrong*; it is usual to claim that the new theory is *not worse* than the others. For example, J. C. Beall (2009, p. 94) says this about Field’s paracomplete position in comparison with his own paraconsistent theory:

The question, in the end, is how to choose between the two given accounts —my account and that of Field’s. The short answer, of course, is that we should choose the right account. The trouble, though, is that, while I reject Field’s proposal and endorse the simple account I’ve laid out in this book, I find myself in the dubious position of enjoying precious little by way of strong objections against Field’s position. As such, I ultimately —though with genuine regret— leave the matter open for future debate.

A similar point is made by some substructural logicians. French (2016, p. 127), for example, rejects the structural rule of reflexivity, and explains the situation in these terms:

We have judged structural reflexivity to be innocent for far too long. Moreover, much like other authors have started to argue about structural contraction and transitivity, we have always had ample reasons to be suspicious of this innocuous seeming principle, and so it is no surprise that once we dig around that such a shady character might be implicated in the paradoxes of self-reference.

This shows again that many pieces in non-classical logic are not supposed to offer the “best theory of validity” but rather an original solution to a puzzle, which can be accepted as a piece of research even if there is no conclusive reason to think that it is strictly better than the other proposals (it is enough to show that it is not worse).

Given this tolerant nature of logical research, it is also important to understand what *logical progress* means in this context. According to my view, progress does not consist in accepting a specific solution to a puzzle; if that were the case, then almost no progress would have been achieved. Rather, it consists in developing more original or sophisticated tools and methods, and on reaching agreements about how to work with them. For example, after Kripke (1975) we know how to prove the non-triviality of a non-classical theory of truth using a fixed-point method; Kripke himself thought this was the major contribution of his theory, as we discussed above. This technique became standard and was also used by many other authors, even by logicians who do not share Kripke’s paracomplete point of view, such as Beall (2009). In other words, Kripke did not make the community agree on the paracomplete solution, but he provided a new technique which could be used by different non-classical logicians. This is the kind of progress that we usually reach while solving semantic paradoxes.

Therefore, even though we often act as if non-classical logics come to revise classical logic, we should be careful about this. In most of the cases, non-classical logics can provide original or illuminating solutions to some important puzzles. Moreover, they make us think more and better about a variety of subjects (including metaphysics, semantics, and linguistics). They introduce technical and conceptual improvements, and new proof methods which may transform (up to a certain extent) the normal activity of scientists. But non-classical logics, when applied to puzzle-solving activities, are not supposed to

replace classical logic as a common language, and as a general theory of reasoning.

3.4. Hybrid perspectives

Most of the work in philosophical logic is developed in a classical meta-language, following the conventions of the puzzle-solving methodology. However, some authors have provided non-classical alternatives. I will try to analyze these developments from the point of view of my theory of logical practice.

Theories such as Priest's dialetheism (2006) are in a more hybrid position between revolutionary and puzzle-solving proposals. Priest proposes a more general view of reality, where "true contradictions" can describe not only semantic but also metaphysical phenomena. However, his general theory is not an exception for the methodology I described: Priest uses his theory to solve philosophical puzzles, he appeals to classical meta-theory, and is particularly concerned about recapturing mathematical reasoning (2006, ch. 8).¹⁴

There are also some non-classical theories of logical validity. For example, Meadows (2014) offers a paraconsistent theory where sentences can be neither valid nor invalid. However, I don't think that those proposals are outside the methodology that I described here. For even though the concept of Validity is non-classical, the meta-meta-language remains classical. For example, in Meadows' theory, Validity can be characterized with a Kripke-like fixed-point construction. This requires classical set theory. These constructions are in fact very similar to the non-classical solutions to the Liar paradox, now replacing truth by validity.

In a similar line, Bacon (2013) developed a "non-classical meta-theory for non-classical logic". In this paper, Bacon offers an algebraic theory of validity, where notions such as *valid* or *provable* may have intermediate values. Clearly, meta-logical notions might have

¹⁴ In a recent paper, Martin (2020) uses the case of Priest's dialetheism in order to argue in favor of abductivism about logic in general. According to Martin's analysis, Priest (2006) argues that dialetheism can solve the Liar and Russell's paradox; but it also has an independent motivation for accepting contradictions, it gives a unified explanation of different phenomena (including some metaphysical puzzles), and it can regain some elements of intuitive mathematical reasoning such as naive set theory. I think, however, that Priest represents one way of theorizing, focused on providing unifying explanations of different phenomena. But as I argued above, the defense of a comprehensive and unifying philosophical point of view is not an essential feature of non-classical approaches.

interesting non-classical descriptions. However, we should discuss how far they go. As in Meadows' theory, the meta-meta-logic of Bacon's approach is classical (see Woods, 2019, pp. 7-8 for more details about this). For example, the algebraic structure for characterizing validity is described using classical logic. This could indicate, according to my view, that this specific approach also belongs to the classically grounded puzzle-solving methodology.

The developments by Zach Weber and colleagues have a more revolutionary nature, for some of them make use of a non-classical meta-logic. Just to take an example: Weber et al. (2016) develop an original theory of non-classical truth-tables, based on paraconsistent set-theory. The theory is *sound and not sound* (p. 10). They also provide (p. 12) a circular proof of non-triviality of set theory which does not look like any other canonical non-triviality proof:

Either naive set theory is trivial or not. If not, we are done. If trivial, then, since this very proof is in naive set theory, it follows that the system is not trivial —since, after all, anything follows, QED.

Some of these results and proposals might be revolutionary. In any case, the development of exceptional research, which does not follow the common rules, is always a possibility. How a piece can become a new and groundbreaking direction in logical research is often hard to anticipate. I would observe that these purely non-classical perspectives are by now minoritarian.

3.5. *Other branches of non-classical research*

In this paper I focused on different non-classical approaches that were proposed as solutions to some specific problems, such as semantic paradoxes. But, as we know, the development of non-classical logic involves much more than that. For example, non-classical logics can be used to represent some semantic or pragmatic features of natural language. They are also useful, if not necessary, for standard philosophical reasoning. For example, discussions about counterfactuals or conditionals usually involve (broadly speaking) non-classical approaches such as modal or conditional logics. Non-classical logics can also be useful to understand philosophical concepts such as “grounding” or “aboutness”.

It would be impossible to enumerate all the applications of non-classical logics to computer science, semantics, linguistics, and

philosophical research in general. Moreover, some logical research is more detached from philosophical concerns: logicians may develop a complete and consistent axiomatic system for a non-classical logic which has no clear philosophical application. Some logical systems might be intrinsically interesting.

In this paper I focused on paradox-solving logics for a clear reason: these approaches are typically framed as “rivals” of classical logic. Trying to understand the structure of a complex logical system with no clear application is a legitimate logical activity; and using a non-classical logic in a specific philosophical debate (for example, for reconstructing “grounding” statements) is also useful and worthwhile. But these avenues of research are not supposed to challenge classical logic as the correct theory of logical validity. They are just not answering the question “which logic is the correct one?”. What I wanted to analyze here is whether the logics that are typically proposed as “rivals” to classical logic represent a genuine challenge for it. I argued that sometimes they do (such as in the case of intuitionism), but most of the times, and more clearly in the discussion about semantic paradoxes, they provide solutions to some logical puzzles under well-established rules. This puzzle-solving activity, far from challenging classical logic in its central role, usually presupposes that it can be used as a common language and-as a general theory of reasoning.

4. Conclusion

In this paper, I argued in favor of a puzzle-solving version of logical anti-exceptionalism. This is not supposed to describe every aspect of logical activity, but it applies to some important discussions such as the solutions to semantic paradoxes. According to my view, in these cases philosophical logicians work with a puzzle-solving methodology. This methodology permits and motivates the development of non-classical logics, but the meta-language must stay classical. In most cases, non-classical logics provide solutions to specific puzzles; but it would be naïve to regard non-classical proposals as possible replacements of classical logic as a common language and as a general theory of reasoning. With the tools we normally use, classical logic cannot be replaced.

The authors I mentioned in the second section were partially right. Williamson was right about the virtues of classical logic. But people do not choose this theory because of its virtues. Classical logic is the fundamental piece of our methodology; we choose classical logic as opera characters choose to sing. Regarding classical and non-classical

logics as merely competing hypotheses is not realistic. And Hjortland was right with respect to solutions to puzzles: non-classical theories can be promising as theoretical contributions, as the history of logic clearly shows. But given the rules of the common methodology, most of them are simply unable to replace classical logic in its central role.

To be clear, I do not want to defend the status quo or to argue that classical logic will never be revised as the main piece of the logical practice in the future. This is always a possibility, but we are not able to see with precision what this revision would be like. This revolutionary kind of revision will not emerge from the typical puzzles that non-classical logicians solve, such as semantic paradoxes.

Finally, we can go back to where we started. Logic is not exceptional: it can be characterized by a scientific methodology. However, at least in some important philosophical discussions, this methodology is not the classical theory of hypothesis testing, but a puzzle-solving activity. Therefore, its core is revisable but not using the tools of its normal activity. Revising classical logic in its fundamental role is a possibility, but only as a revolution, not as part of our regular puzzle-solving practice.

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