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SECRECY, CONTENT, AND QUANTIFICATION

Secretos, contenido y cuantificación

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Abstract

While participating in a symposium on Dave Ripley's forthcoming book *Uncut,* I had proposed that employing a strict-tolerant interpretation of the *weak* Kleene matrices provided a content-theoretical conception of the bounds of conversational norms that enjoyed advantages over Ripley's use of the *strong* Kleene matrices. During discussion, I used the case of sentences that are taken to be out-of-bounds for being *secrets* as an example of a case in which the setting of conversational bounds in practice diverged from the account championed by Ripley. In this paper, I consider an objection that my treatment of quantifiers was mistaken insofar as the confidentiality of a sentence $\varphi(t)$ may not lift to the sentence $\exists x\varphi(x)$ and draw from this objection that neither the strong nor the weak Kleene interpretation of quantifiers suffices, but that a novel interpretation may do so.

Key words: Weak Kleene Logic; Strict-Tolerant Logic; Immune Logic; Quantifiers.

Resumen

Mientras participaba en un simposio sobre el libro de David Ripley *Uncut*, de próxima aparición, sugerí que emplear una interpretación tolerante estricta de las matrices de Kleene débiles proporcionaba una concepción teórica del contenido de los límites de las normas conversacionales que tenía ciertas ventajas sobre la propuesta de Ripley, la cual está basada en matrices de Kleene fuerte. Durante la discusión, utilicé el caso de los secretos como ejemplos de oraciones donde los límites conversacionales en la práctica divergen de la explicación defendida por Ripley. En este artículo, considero la objeción de que mi tratamiento de los cuantificadores era erróneo en virtud de que la confidencialidad de una oración $\varphi(t)$ puede no afectar la oración $\exists x\varphi(x)$ y extraigo de esta objeción la conclusión de que en lo que respecta a los cuantificadores, ni la interpretación de fuerte de Kleene ni la débil son suficientes, pero una interpretación novedosa podría serlo.

Palabras clave: Lógica Kleene débil, Lógica estricta-tolerante, Lógica inmune, Cuantificadores.

1. Introduction

This paper aims to elaborate on some issues that came up during the discussion of a contribution I had made to the symposium on Dave Ripley's forthcoming book *Uncut* hosted at the Argentinean Society of Philosophical Analysis. During the symposium, I discussed the matter of *bounds-setting* and objected to the idea that the types of positions that are in- or out-of-bounds are determined by purely *veridical* concerns.

The bounds consequence interpretation of a sequent follows Greg Restall's work in Restall (2005) in interpreting a sequent $[\Gamma \vdash \Delta]$ as a *position* that one might take in a discourse, namely, the position in which each formula in Γ is asserted and each formula in Δ is rejected.

> Let a *position* be some pair $\langle \Gamma, \Delta \rangle$ of assertions and denials. These might be, for the example, the assertions and denials actually made by one party to a particular debate. The position $\langle \Gamma, \Delta \rangle$ asserts each thing in Γ and denies each thing in Δ. Some positions are *in bounds*, and some are *out of bounds*. (Ripley, 2013, p. 141)

The derivability of a position $[\Gamma \vdash \Delta]$ is construed as an affirmation that taking that position is *out-of-bounds*, that is, to adopt this position is to flout conversational norms in some way. For example, that the position $[\varphi \wedge \psi \vdash \varphi]$ is demonstrable corresponds to the fact that it is out-of-bounds to simultaneously assert a conjunction while rejecting one of its conjuncts.

The accounts of Restall (2005) and Ripley (2013) are willfully agnostic concerning *how* the bounds of discourse are set, indicating that there may be multiple notions of coherence of a position for a language. In the book (Ripley, 2018) on which the symposium was held, Ripley maintains that the features of conversational practices play a critical role in the determination of bounds and, it follows, in the determination of consequence itself:

> [T]here is nothing interestingly *logical* about the notion of bounds in play here. The notion we can find playing a role in our conversational practices is a *material* one. (Ripley, 2018, p. 12)

Despite this, both Restall and Ripley seem to assume that the features that make a position coherent or in-bounds are *veridical* in nature, that is, that the truth or falsity of the constituents of a position is sufficient to determine whether or not a position is in-bounds. We might think of this as the *veridical conception of bounds-setting.*

My objection to this notion of bounds-setting follows from a simple observation that although veridical concerns clearly play *some* role in determining the bounds of appropriate positions in common discourse, an equally important dimension is that of *content*. This is to say that there are sentences that, despite their truth, are rejected as out-of-bounds due to features of their content. Examples are easy to come up with; suppose that φ is an assertoric statement that happens to contain the type of horrific profanity that makes any well-mannered individual blush. If we restrict our attention to the realm of the veridical, then insofar as φ is assertoric and expresses a proposition—if not, perhaps, a *polite* proposition——it has a truth-value, whence the complex $\varphi \vee \neg\varphi$ should be considered true. Nevertheless, the same content-theoretic properties of φ that cause the position $[\varphi \vdash]$ (i.e., the assertion of φ) to be out-of-bounds will lift to the complex $\varphi \vee \neg\varphi$ —i.e., the tautologousness of this instance of excluded middle is insufficient to cleanse the complex of its profanity—and, hence, the mere fact of its truth is insufficient to make it in-bounds.

In other words, while I agree with Ripley and his collaborators that there exist sentences φ for which both positions $[\varphi \vdash]$ and $[\vdash \varphi]$ are out-of-bounds, I believe that the class of such sentences is broader than Restall or Ripley would allow. Characteristic examples for Ripley include the liar sentence λ, in which case these positions are out-of-bounds inasmuch as one can both the assertion and rejection of λ lead to incoherence. So far, so good. But it strikes me as obvious that our conversational norms—and the bounds that we thereby set—reject myriad types of truths as out-of-bounds on their faces. I suspect (and, for their sakes, I *hope*) that even those who might resist the notion that conversational bounds are so constrained nevertheless police their language at weddings, commencements, banquets, and so forth, thereby *tacitly* admitting that there exist such extra-veridical bounds on discourse.

There are semantical consequences that follow from broadening the class of out-of-bounds positions. That positions $[\varphi \vdash]$ and $[\vdash \varphi]$ are both out-of-bounds holds when the content of φ is sufficient to reject its utterance in discourse. We can illustrate what is at stake for the notion of bounds consequences by considering the following rule:

$$\frac{[\varphi \vdash] \qquad [\vdash \varphi]}{[\varphi \vee \psi \vdash]}$$

On the standard bounds consequence reading, such a rule is not admissible. Even the case of a malignant sentence like λ can be "cleansed" by disjoining it with a true sentence ψ, e.g., it is not out-of-bounds to assert "Either the liar sentence is true or the Earth is round." But if both the assertion and rejection of φ is out-of-bounds due to its *content*, then for any ψ whatsoever, to assert $\varphi \vee \psi$ will, too, be out-of-bounds.

During the symposium, I made the following remarks as providing further evidence that conversational bounds are governed in part by content-theoretic considerations:

> A further colloquial example is the case in which utterances are deemed out-of-bounds due to the confidential nature of some element of their content. In cases in which φ is a secret with which one has been entrusted, whether or not one has betrayed one's responsibilities is not equivalent to the question of whether or not one has committed oneself to the truth of φ. Rather, the mere act of making φ salient places one outside the bounds that have been set for such a speaker. For example, if the encryption key to Y Corporation's files is a binary string σ, to send an email message to the CTO of Z Industries saying "Either the Earth is round or the encryption key to Y Corporation is σ" is no less an act of corporate espionage than to send a message unequivocally asserting that the encryption key is σ. Similar considerations hold with respect to matters of insider trading, the identities of undercover assets, military landing zones, and so forth.

After giving these remarks, I received an objection that is worth considering in more detail, as it bears on the matter of how quantifiers should be best interpreted in a weak Kleene setting. I will first try to outline the type of case to which the objection is directed in more detail, allowing me to more satisfactorily outline the objection. I will then outline some formal details before discussing my response to the objection and some of its consequences.

2. The Secrecy Case and its Objection

The secrecy case is this: Suppose that one has been entrusted with a secret of some sort, say, the password to a highly sensitive computer network. Suppose, moreover, that the password is "marriedcowboy" and let ψ be the sentence:

(1) The network password is ‘marriedcowboy’.

Then this out-of-boundedness due to secrecy seems to provide a case of a semantic property that renders ψ out-of-bounds that lifts from the atomic assertion through complexes containing it.

For example, the formula $\neg\psi$, i.e., the sentence:

(2) The network password is *not* ‘marriedcowboy’.

would be similarly out-of-bounds. For a network administrator for the network to go on a public forum and publicly declare $\neg\psi$ would be an invitation for others to try their luck with ‘marriedcowboy’. Despite not having veridically acknowledged the password, the act of denying that this is the password would be sufficient to open the administrator to liability for having made the actual password salient. And, it seems to me, for that administrator to make the defense that one can’t reveal the truth of a sentence by *denying* it seems weak, as this assertion would be an open invitation to use this very password. (In such a case, one might reason “the administrator doth protest too much, methinks.”) Thus, $\neg\psi$ is out-of-bounds not due to any veridical concern, but for a content-theoretic one.

This type of consideration appears to lift through conjunctions and disjunctions as well. Clearly, a conjunctive assertion including **(1)** would be out-of-bounds inasmuch as it asserts the truth of that conjunct. The content element becomes more clear in the case of disjunction, however. To utter a sentence such as:

(3) Either the network password is ‘marriedcowboy’ or the network password is ‘uglyduckling’.

would allow would-be network penetrators guaranteed access to the system within two tries. Given the finitary nature of a disjunction, a disjunction including (1) of *any* length would still suggest to a penetrator enough information to gain access to the network within a finite number of attempts. If, then, a property like secrecy is sufficient to place a simple straightforwardly and uncontroversially out-of-bounds for an agent, this property appears to lift to complexes in which it appears.

The objection is simply this: Although there is some plausibility to the suggestion that a “secret” disjunct may (for possibly pragmatic reasons) leave a disjunction out-of-bounds, the plausibility stops at the level of quantification. For if quantifiers are given the interpretation of (possibly infinitary) conjunctions and disjunctions, and the content of (e.g.) a disjunction includes the content of each disjunct, then the content of "There exists a network password" should include the content of the sentence "The network password is σ" for every possible password σ.

To make this more precise, recall the following rule:

$$\frac{[\varphi \vdash] \qquad [\vdash \varphi]}{[\varphi \vee \psi \vdash]}$$

The admissibility of this rule appears to correspond to setting bounds according to content-theoretic considerations. If, in other words, it is out-of-bounds to both assert and to reject φ, then merely "padding" φ with a disjunct is insufficient to bring the complex back in-bounds. Now, the line of reasoning at the heart of the objection suggests that insofar as the existential quantifier is *au fond* an infinitary disjunction, the kinship between the two entails that someone endorsing a content-theoretic dimension to bounds-setting should be committed to the admissibility of a rule like:

$$\frac{[\varphi(t) \vdash] \qquad [\vdash \varphi(t)]}{[\exists x \varphi(x) \vdash]}$$

But, goes the objection, despite the fact that ψ is considered out-of-bounds by hypothesis, we should hardly think that for a network administrator to concede that

(4) There exists a network password.

would be to reveal any secrets with which he or she was entrusted. After all, not only does σ not appear in the sentence—and therefore is not made salient to possible network infiltrators—but the assertion itself is trivial for any network administrator. The very identity of an individual as a network administrator comes, in other words, with the implicit knowledge that there is *some* password for access to the network that he or she oversees.

I think that it would be hasty to provide the syntactical rejoinder that although the password σ *appears* in the disjunction (3), the string does not *appear* in the quantified sentence (4). For what is at stake is not that the string appears, but rather, that the string is *recoverable* or *made salient* by the sentence. In the case of the former consideration, it seems that if our network administrator were, in fact, a married cowboy, it could well be in-bounds to express this fact despite the syntactic convergence with the password itself. For the latter consideration, despite the fact that 'marriedcowboy' does not appear in the following:

(5) The network password is 'nbssjfedpxcpz'.

to express the proposition would likely cost the administrator his or her job, inasmuch as 'nbssjfedpxcpz' is easily recognizable as the result of the Caesar cipher with a right shift of one applied to the string 'marriedcowboy'. In other words, the mere syntactical appearance of the password σ is neither necessary nor sufficient to place a position out-of-bounds due to its content.

It is important to notice that the objection assumes an important presupposition, namely, that the symmetry between the sentential connectives and the quantifiers is good in itself and ought to be preserved at all costs. Certainly, the symmetry between, e.g., disjunction and the existential quantifier is attractive, both semantically and from an interpretative standpoint. There is a kinship between how each type of operator expresses De Morgan's laws or distributivity and we readily identify both $\neg(\varphi \vee \psi) \leftrightarrow (\neg\varphi \wedge \neg\psi)$ and $\neg\exists x\varphi(x) \leftrightarrow \forall x\neg\varphi(x)$ as sides of the same medal.

The appropriate response, I think, is that the symmetries between the binary connectives conjunction and disjunction and the universal and the existential quantifiers must be broken. This requires a concession on my part, to some degree. To illustrate, let us first examine the formal elements of what I had presented.

3. Content and Quantification

First, let us look at how the sequent calculus and *strict-tolerant* logic championed by Ripley operate. In Ripley's *Kleene-Kripke* models of Ripley (2013)—and the most frequently encountered approaches to liar sentences, like those of Field or Priest—the semantic behavior of a paradoxical statement like λ follows the *strong Kleene matrices* as described below:

¬	
1	0
½	½
0	1

∧	1	½	0
1	1	½	0
½	½	½	0
0	0	0	0

∨	1	½	0
1	1	1	1
½	1	½	½
0	1	½	0

The strong Kleene interpretation of the quantifiers is as follows, where the functions are operations on sets of truth-values:

$$\forall(X)=\min(X) \qquad \exists(X)=\max(X)$$

Bounds consequence, as defined by Ripley, mirrors these matrices. Hence, if the position $[p \vdash q]$ is not out-of-bounds, $[\lambda \vee p \vdash q]$, too, remains in-bounds. More formally, a Kleene-Kripke model is defined in Ripley (2013) as follows:

Definition 1. *A Kleene-Kripke model for a language $\mathcal{L}$ is a pair $\langle D, I\rangle$ where D is a domain of elements such that $\mathcal{L} \subseteq D$ and I is an interpretation such that:*

- *for a term t, $I(t) \in D$*
- *where $\ulcorner \varphi \urcorner$ is a distinguished name for φ, $I(\ulcorner \varphi \urcorner) = \varphi$*
- *for an n-ary predicate R, $I(R)$ maps n-tuples from D^n* to $\{0,½,1\}$
- *for atoms $P(\vec{t})$, $I(P(\vec{t})) = I(P)(I(\vec{t}))$*
- *sentential connectives and quantifiers are given the strong Kleene interpretation*
- *$I(T\ulcorner \varphi \urcorner) = I(\varphi)$ for all formulae φ*

Then we define strict-tolerant validity as follows:

Definition 2. *An ST-counterexample to a sequent $\Gamma \vdash \Delta$ is a Kleene-Kripke model such that $I''(\Gamma)=\{1\}$ and $I''\Delta=\{0\}$. The inference $\Gamma \vdash^{ST} \Delta$ is ST-valid if there are no ST-counterexamples.*

The sequent calculus described in Ripley (2013) is sound and complete with respect to the consequence relation induced by the strong ST framework and, therefore, I take the strong ST semantics to correspond to the veridical conception of bounds-setting.

In the symposium, I championed the use of the *weak* Kleene matrices and that the ST-type interpretation (a *weak* ST framework) would correspond to a content-theoretic conception of bounds-setting. Instances of logics employing the weak Kleene such as the classical or "internal" fragments of the nonsense logics described by Bochvar (1938) or Halldén (1948). The weak Kleene matrices have a familiar representation as follows:

¬	
1	0
½	½
0	1

∧	1	½	0
1	1	½	0
½	½	½	½
0	0	½	0

∨	1	½	0
1	1	½	1
½	½	½	½
0	1	½	0

These matrices are notable for the property of *infectiousness* of the third value, i.e., that the appearance of a subformula evaluated as ½ in a complex entails that the complex itself is evaluated as ½.

The commonly-accepted quantificational correlates to the three-valued weak Kleene matrices are described in Malinowski (2008) as follows, again, as operations on sets of truth-values:

$$\forall(X) = \begin{cases} 1 & \text{if } X = \{1\} \\ \frac{1}{2} & \text{if } \frac{1}{2} \in X \\ 0 & \text{otherwise} \end{cases} \qquad \exists(X) = \begin{cases} 1 & \text{if } 1 \in X \text{ and } \frac{1}{2} \notin X \\ \frac{1}{2} & \text{if } \frac{1}{2} \in X \\ 0 & \text{otherwise} \end{cases}$$

Now, recall Ripley's Kleene-Kripke models of Definition 1 and consider the following definition:

Definition 3. *A weak Kleene-Kripke model is defined analogously to Definition 1 with the exception that sentential connectives and quantifiers are interpreted by the weak Kleene matrices.*

And let us provide a similar modification to validity along the lines of Definition 2:

Definition 4. *A weak ST-counterexample to a sequent $\Gamma \vdash \Delta$ is a weak Kleene-Kripke model for which $I''\Gamma=\{1\}$ and $I''\Delta=\{0\}$. We say that $\Gamma \vDash^\star \Delta$ is weakly ST-valid if there are no weak ST-counterexamples.*

If we provide a bounds consequence reading to $\vDash^\star$, we can continue to understand the validity of a position as the suggestion that it is out-of-bounds to strictly assert each member of Γ while strictly denying each member of Δ. As a historical note, the strict-tolerant interpretation of the weak Kleene matrices is implicit in Fabrice Correia (2002), in which a consequence relation $\Vvdash_C$ is considered.[1]

Given the relatively extreme behavior of the internal fragments of Bochvar's and Halldén's logics, it might be surprising to learn that the strict-tolerant reading of the weak Kleene matrices induces classical logic when the standard connectives and quantifiers are considered. Just as Ripley (2013) shows us that ST-validity coincides with classical validity, we are able to establish the following observation. Call a formula in which no instances of the truth predicate or corner quotes appear "classical." Then:

[1] I am grateful to Damián Szmuc for pointing this out to me.

Observation 1. *When Γ and Δ contain only classical formulae,*

$$\Gamma \vDash^{\star} \Delta \textit{ iff } \Gamma \vDash_{CL} \Delta$$

Proof. For left-to-right, note that classical valuations are a proper subset of three-valued valuations. Hence, if $\Gamma \vDash^{\star} \Delta$ then all classical valuations in which all $A \in \Gamma$ are assigned the value of 1 have some $B \in \Delta$ for which $v(B) \neq 0$.

For right-to-left, suppose that $\Gamma \vDash_{CL} \Delta$ and let v be an arbitrary three-valued valuation. There are two cases to consider: The case in which v maps all atoms appearing in $\Gamma \cup \Delta$ to an element of {0,1} and those in which v maps some such atom to ½. In the first case, v restricted to the formulae in $\Gamma \cup \Delta$ is for all intents and purposes a classical valuation, whence if $v(\varphi)=1$ for all $\varphi \in \Gamma$, then $v(\psi) \in \{½,1\}$ for some $\psi \in \Delta$. In the second case, then by the infectious nature of ½, there is a $\xi \in \Gamma \cup \Delta$ for which $v(\xi)=½$. Of course, if $\xi \in \Gamma$ then not all formulae in Γ take the value 1 and if $\xi \in \Delta$ then some formula in Δ is evaluated as a member of {½,1}, whence the validity condition is alternatively vacuously or trivially satisfied. □

Observation 2. *In any sound and complete sequent calculus corresponding to the weak ST framework, the following rule is admissible:*

$$\frac{[\varphi \vdash\ ] \qquad [\ \vdash \varphi]}{[\varphi \vee \psi \vdash\ ]}$$

Proof. Suppose that $[\varphi \vdash]$ and $[\vdash \varphi]$ are both derivable. Given the hypothesized completeness of the sequent calculus in which the sequents were derived, this means that there are neither models in which φ receives a value of 1 nor models in which φ receives a value of 0. By the "infectiousness" of the weak Kleene matrices, compositionality makes it easy to confirm that *every* formula has a model in which it is assigned a value of ½, however. It follows that in all models φ will receive the value ½. Consulting the truth tables for the weak Kleene matrices confirms that in all models $\varphi \vee \psi$ will likewise receive the value ½. Thus, there are no models in which $\varphi \vee \psi$ receives a value of 1 and, given the supposed soundness of the sequent calculus, the position $[\varphi \vee \psi \vdash]$ will be derivable. □

Now, the objection I had received does compel me to walk back the appropriateness of the weak ST framework as I had described it for the following reason:

Observation 3. *In any sound and complete sequent calculus corresponding to the weak ST framework, the following rule is admissible:*

$$\frac{[\varphi(t) \vdash] \quad [\vdash \varphi(t)]}{[\exists x \varphi(x) \vdash]}$$

Proof. As before, when $[\varphi (t) \vdash]$ and $[\vdash \varphi (t)]$ are both derivable, this means that in no model does the interpretation of $\varphi (x)$ map a term to a value 0 or 1 and that in *some* model, its interpretation maps at least one term to the value ½. This means that the class of models in which $\varphi(t)$ takes the value ½ is precisely the class of models in which $\exists x\varphi(x)$ is assigned ½. Consequently, there are no models in which $\exists x\varphi(x)$ is assigned the value 1. Thus, given the supposed soundness of the sequent calculus, the position $[\exists x\varphi(x)\vdash]$ will be derivable. □

In short, the objection I had received at the symposium meant that the weak ST semantics are, in fact, inappropriate for a content-theoretical conception of bounds-setting.

4. Discussion

With the foregoing in mind, I think that the objection has a great deal of merit, if not against a content-sensitive account of bounds, at least insofar as a prompt to reevaluate the relationship between the connectives and the quantifiers. And, upon examination, it strikes me that the harmony between the sentential connectives and the quantifiers is already questionable in the context of logics of nonsense, even before one considers a strict-tolerant formulation.

Consider, for example, one of the prototypical applications described by Halldén, that of category mistakes. Following remarks of the positivists (such as Carnap (1931)), Halldén suggests that a predicate P has a range of elements to which it may be meaningfully applied, i.e., a set of constants t for which $P(t)$ is not a category mistake. If one accepts that there are indeed category mistakes—and that such sentences are meaningless—then taking stock of familiar predicates suggests that nearly all predicates have a non-universal range of significance. Intuitively, *evenness* meaningfully applies only to natural numbers, the property of *being a general* meaningfully applies only to persons, *truth* applies only to sentences or propositions, and so forth. Logics of nonsense provide one way of making these intuitions precise.

If this is the case—or even if a hefty minority of predicates are limited in this way—then using the weak Kleene quantifiers entail that *it is always meaningless to quantify into these predicates.* Suppose, for example, that one wishes to express the apparently uncontroversially meaningful sentence:

(6) All natural numbers are prime.

N.b. that I am not suggesting that this sentence is *true*, but merely that (6) appears to avoid the label of "category mistake" effortlessly and uncontroversially.

However, if we play along with the Carnapian assumption that there exist category mistakes, then *the* archetypal example of a category mistake is the following:

(7) Caesar is prime.

Now, if we *were* to take the weak Kleene quantifiers outlined by Malinowski as appropriate to the task of a logic of nonsense, then because the predicate "...is a prime," when applied to the constant Caesar, yields nonsense (i.e., its interpretation maps Caesar to the value ½), *so, too, must the quantified sentence be nonsense.* So "infectiousness" in the case of quantifiers already seems suspect.

Yet, if one is to satisfy the goal of the logics of nonsense (or, by extension, a content-theoretical conception of bounds-setting), the more familiar strong Kleene connectives seem just as suspect. Consider the case of a number-theoretic statement such as:

(8) There exists a natural number that is divisible by zero.

It strikes me that this sentence is false *simpliciter*. It cannot be *true*, as we all were taught in primary school. Nor, however, is it on its face a category mistake; the sentence takes a number-theoretic property (i.e., the predicate "...is divisible by zero") and asserts that it holds of at least one member of a class of objects clearly within its range of significance.

Nevertheless, there is a further problematic Carnapian sentence to consider:

(9) Caesar is divisible by zero.

If this sentence is supposed to be a category mistake, the class of substitutions includes sentences of two kinds: Those that are false (i.e., those that are evaluated as 0) and those that are category mistakes (i.e., those that are evaluated as ½). The path down which the strong Kleene quantifiers lead us, then, ends with the sentence "There exists a natural number that is divisible by zero" taking the *maximum* value from the set {½,0}, entailing that the existential sentence, too, is meaningless. The strong Kleene quantifiers, then, are no balm of Gilead.

So, the objection I had received at the symposium ultimately leads us to reconsider the account of quantification appropriate to this task. The strict-tolerant reading of the strong Kleene quantifiers appears to induce the veridical conception of bounds consequence, while the strict-tolerant reading of the weak Kleene quantifiers seems too permissive. In response to this dilemma, I would like to propose the following account of quantification:

$$\forall(X) = \begin{cases} 1 & \text{if } 0 \notin X \text{ and } 1 \in X \\ \frac{1}{2} & \text{if } X = \{\frac{1}{2}\} \\ 0 & \text{otherwise} \end{cases} \qquad \exists(X) = \begin{cases} 1 & \text{if } 1 \in X \\ \frac{1}{2} & \text{if } X = \{\frac{1}{2}\} \\ 0 & \text{otherwise} \end{cases}$$

Semantically, these quantifiers seem to be the infinitary analogues of the following matrices for conjunction and disjunction, respectively:

∧	1	½	0
1	1	0	0
½	0	½	0
0	0	0	0

∨	1	½	0
1	1	1	1
½	1	½	0
0	1	0	0

The interpretations of the quantifiers described above enjoy many of the symmetries with conjunction and disjunction; it is easy to confirm that De Morgan's laws and distribution still hold, for example. They are recognizably universal and existential quantification, respectively, and have appeared as such in Carnielli et al. (2000). Because these matrices have been considered by Damián Szmuc and Bruno Da Re (2021) as enjoying a property of *immunity* (in contrast to the weak Kleene property of *infectiousness*), we might refer to these quantifiers as the *immune Kleene quantifiers*.

And they seem to harmonize with the motivations of the progenitors of logics of nonsense—a primary source of motivation for the weak ST framework that I had described. Moreover, let us call the weak Kleene matrices for negation, conjunction, and disjunction coupled with the immune Kleene quantifiers quantifiers—in which the infectious connectives are joined with the immune quantifiers—the *variegated* Kleene matrices.

The serviceability of the matrices I have suggested against the objection will be tested by how the strict-tolerant interpretation fares. So let us consider several definitions to generate a corresponding variegated strict-tolerant logic.

Definition 5. *A variegated Kleene-Kripke model is defined analogously to Definition 1 with the exception that sentential connectives are interpreted by the weak Kleene matrices and the quantifiers are interpreted by the immune Kleene matrices.*

As before, we consider a modification to the notion of validity expressed in Definition 2:

Definition 6. *A variegated ST-counterexample to a sequent $\Gamma \vdash \Delta$ is a variegated Kleene-Kripke model for which $I''\Gamma = \{1\}$ and $I''\Delta = \{0\}$. We say that $\Gamma \vDash^{\star} \Delta$ is variegatedly ST-valid if there are no variegated ST-counterexamples.*

This *variegated* ST framework appears to enjoy a harmony with the content-theoretical conception of bounds-setting while safely avoiding the perils of the objection.

We, for example, are able to accommodate the content-theoretical considerations that lead us to infer from the out-of-boundedness of (1) the consequent out-of-boundedness of (3). This can be seen from the following observation:

Observation 4. *In any sound and complete sequent calculus corresponding to the variegated ST framework, the following rule is admissible:*

$$\frac{[\varphi \vdash\ ] \qquad [\ \vdash \varphi]}{[\varphi \vee \psi \vdash\ ]}$$

Proof. Identical to the proof for the weak ST case. □

However, we are able to also avoid the objection that we are committed to the out-of-boundedness of (4). This is represented by the *inadmissibility* of the following rule:

Observation 5. *In any sound and complete sequent calculus corresponding to the variegated ST framework, the following rule is* not *admissible:*

$$\frac{[\varphi(t) \vdash] \qquad [\vdash \varphi(t)]}{[\exists x \varphi(x) \vdash]}$$

Proof. As in earlier proofs, completeness of the hypothesized sequent calculus entails that in all models $\varphi(t)$ receives a value of ½. We can draw from Ripley's motivation the case of the liar sentence λ, which is equivalent to $\neg T(\ulcorner \lambda \urcorner)$, where T is a truth predicate. But the fact that the unsaturated formula $\neg T(x)$ maps *something* (i.e., λ) to ½ in all models does not mean that it maps *all* arguments to ½. Indeed, by the way in which Ripley (2013) sets up T, the formula $\neg T(\ulcorner x \neq x \urcorner)$ will be evaluated as 1. Hence, by the immune quantifiers introduced above, there are models in which $\exists x \neg T(x)$ is evaluated as 1, providing the variegated ST counterexample needed to show that the position $[\exists x \varphi(x) \vdash]$ is not derivable from the assumptions. □

In these immune interpretations of the quantifiers, then, we have an account of quantification that preserves much of the symmetry with conjunction and disjunction, addresses a deficiency with the weak Kleene interpretation in the field of logics of nonsense, and suggests an appropriate strict-tolerant semantics for a notion of bounds consequence that is attentive to the content-theoretical dimensions of the guidelines for our discourses.

5. Concluding Remarks

There is clearly a great deal of work remaining in the analysis of the suitability of the above-described quantifiers and I should like to return to this matter in future work.

For example, there still exists no demonstrably sound and complete sequent calculus for the weak ST framework, much less the variegated ST framework I have described in this paper. During the symposium, I had made the following conjecture:

Conjecture 1. *The addition of the following two rules to Ripley's sequent calculus **CL** axiomatizes strict-tolerant consequence on the weak Kleene matrices:*

$$\vee L'\colon \frac{[\Gamma, \varphi_i \vdash \Delta] \qquad [\Gamma \vdash \varphi_i, \Delta]}{[\Gamma, \varphi_0 \vee \varphi_1 \vdash \Delta]} \; \textit{for } i \in \{0,1\}$$

$$\wedge R'\colon \frac{[\Gamma, \varphi_i \vdash \Delta] \qquad [\Gamma \vdash \varphi_i, \Delta]}{[\Gamma \vdash \varphi_0 \wedge \varphi_1, \Delta]} \; \textit{for } i \in \{0,1\}$$

While this may hold in the propositional case, it certainly omits any difference with respect to the quantifiers. Finding ways to modify the sequent calculus **CL** of Ripley (2013) to accommodate both the weak and the variegated ST frameworks remains to be done.

Moreover, the objection in many ways reduces to the question of whether the content of a formula $\varphi(t)$ is *part* of the formula $\exists x\varphi(x)$. There are clearly parallels between quantification in the ST family, quantification in logics of nonsense, and quantification in the field of *containment logics*. The work of William Parry (1968) outlines a case for a logic in which implication is understood as the *containment* of the meaning (including the content) of a consequent within the meaning of an antecedent proposition. Parry's work, however, fell short of providing an account of the content of quantified sentences. Many cues exist, such as the work of Charles Daniels' *story semantics* (1986), in which the notion of a *cast* of a story informs the content of an intra-story assertion of a quantified sentence. Similarly, the discussion Kit Fine's semantics for a modified version of Parry's logic in Fine (1986) includes a few relevant remarks concerning the content of a quantified formula:

> [On one] account, the content of $\forall x A(x)$ is the intersection of the contents of $A(a)$ for a any name of an object in the domain; while on [an opposing] account, the content of $\forall x A(x)$ is the union of the contents of all such $A(a)$. The first account seems to be more natural, for in order to understand $\forall x A(x)$ I need not know (or, at least, possess names for) the objects in the domain of the quantifier. (Fine, 1986, p. 178)

It seems that the account of content implicit in the weak Kleene quantifiers shares similarities to the second of the options Fine describes while the strong Kleene quantifiers—employed in the veridical conception of bounds consequence—are ignorant of matters of content. Whether or not—and to what extent—the quantifiers I have described accord with the first of Fine's accounts is also worth exploring, but will be left for a future date as well.

Finally, several observations passed on to me by reviewers of this paper should be taken into account in further development of the notion of the immune quantifiers we have discussed. For one, it should be noted that the immune quantifiers may not share the utility of the weak or strong Kleene quantifiers in fixed-point theories of truth inasmuch as monotonicity may fail during the construction. For example, suppose that there exist only two terms s and t and that at an initial stage, $R(s)$ is assigned a value of ½ and $R(t)$ is assigned a value of 0. Then on the immune reading, $\exists x R(x)$ is evaluated as 0. But if at a subsequent stage, $R(s)$ is assigned a value of 1, then $\exists x R(x)$ will then be assigned a value of 1, showing that monotonicity fails. Whether these immune quantifiers (and the corresponding immune connectives) have any remaining utility in such contexts despite the failure of monotonicity is left open. Moreover, it should be stressed that the inferences we have considered for the variegated Kleene interpretation are tightly bound up with the strict-tolerant interpretation. If the consequence relation is interpreted differently—if, e.g., we consider the matrices and quantifiers in a tolerant-strict interpretation—then the admissible rules will differ significantly. Again, studying these additional variations on such consequence relations is left open for the time being.

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