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ON A BOHMIAN STRUCTURAL APPROACH WITHOUT SPACE FUNDAMENTALITY

Acerca de una interpretación Bohmiana estructural sin fundamentalidad respecto al espacio

Jorge Manero [a]
https://orcid.org/0000-0003-4588-5132
jorge.manero@filosoficas.unam.mx

[a] Instituto de Investigaciones Filosóficas, Universidad Nacional Autónoma de México, Mexico City, Mexico.

Abstract

In recent years, there has been a controversial debate in the literature regarding the choice among many interpretations of quantum mechanics. Based on certain realist desiderata, this debate has been partly addressed in the context of the choice between two opposite views about the mathematical space in which the fundamental ontology of this theory lives. Indeed, some scholars advocate a worldview in which this ontology lives in the abstract high-dimensional space, whilst others advocate a three-dimensional worldview, according to which the ontology lives in the physical space of our everyday experience. In this contribution, I will critically evaluate the tenability of solving the resulting underdetermination between the three/high-dimensional worldviews in the context of one particular quantum theory: the Bohmian approach. In so doing, I will be following two strategies: firstly, I will undermine the assumption that a set of theoretical or metaphysical virtues blocks the underdetermination problem associated with the mathematical space in which the fundamental Bohmian ontology lives; and secondly, I will propose a structural Bohmian interpretation through the use of dynamical symmetries (symplectic Lie groups), according to which the three/high-dimensional space distinction is just apparent as both spaces are grounded by the same dynamical, group-theoretic structure.

Key words: Metaphysics of Quantum Mechanics; Bohm's Theory; Structural Realism; Underdetermination; Lie Group Theory.

Resumen

En los últimos años se ha desarrollado un polémico debate respecto a la elección entre las muchas interpretaciones de la mecánica cuántica disponibles en la literatura. Con base en ciertos criterios realistas, este debate se ha abordado en el contexto de la elección entre dos visiones opuestas sobre el espacio matemático en el que vive la ontología fundamental de esta teoría. En efecto, varios académicos han defendido una visión en la que esta ontología vive en un espacio abstracto de alta dimensión, mientras que otros abogan por una visión tridimensional, según la cual la ontología vive en el espacio físico de nuestra experiencia cotidiana. En esta contribución, evaluaré críticamente la posibilidad de resolver la subdeterminación resultante entre la visión tridimensional y la de alta dimensión en el contexto de una teoría cuántica particular: el enfoque Bohmiano. Al hacerlo, seguiré dos estrategias: en primer lugar, socavaré la suposición de que un conjunto de virtudes teóricas o metafísicas bloquea el problema de la subdeterminación asociado con el espacio matemático en el que vive la ontología bohmiana fundamental; y en segundo lugar, propondré una interpretación Bohmiana estructural mediante el uso de simetrías dinámicas (grupos de Lie simplécticos), según la cual la distinción entre el espacio tridimensional y el de alta dimensión es solo aparente, ya que ambos espacios dependen fundamentalmente de la misma estructura dinámica.

Palabras clave: Metafísica de la mecánica cuántica; Teoría Bohmiana; Realismo estructural; Subdeterminación; Teoría de grupos de Lie.

1. Introduction

According to many scholars, the Bohmian theory (BQM) is an empirically equivalent formulation of quantum mechanics (QM) capable to be interpreted realistically. The task of the realist in this context is basically of elaborating, clarifying and extending the objective correspondence that is hypothesised to hold between BQM and the aspects of the actual world that this theory approximately characterises —if the relevant sentences of BQM are to be held as approximately true and its theoretical terms as referring. Unfortunately, this task is not as straightforward as it may seem, because even if there is an underlying formulation that serves to present BQM in terms of a set of minimal principles and postulates, there are different and even incompatible interpretations of this formulation available, leading to a contrastive form of strong underdetermination.[1] In particular, one may recognise two opposed 'Bohmian schools' (considering important variants

[1] As insisted by Laudan (1990), underdetermination comes in a wide variety of strengths depending on the scope attributed to the confirming evidence —apart from other features associated with the corresponding hypotheses. As regards contrastive underdetermination (arising from empirical equivalence), some scholars have claimed that it appears in a weak and strong version (Bonk, 2008). The weak version, on the one hand, is the most common situation in which two or more theories only share the same set of evidence at some given time, but not necessarily at some later time. In this case, underdetermined theories might stop sharing a common evidence due to certain changes occurring at the level of their hypotheses or at their empirical level, where new evidence might appear at a later time —requiring the extension of their domains. The strong version, on the other hand, involves the claim that for every theory there is always at least one theory which is empirically equivalent and shares all possible sets of past, present and future evidence. It should be noted, though, that in this contribution, we shall talk about the strong version of contrastive underdetermination.

among them) amply discussed in the literature: the *neo-Newtonian approach* advocated by Bohm (1952a), Bohm and Hiley (1993), Holland (1993) and the *guiding approach* elaborated by Bell (1971), Dürr et al. (1992), Valentini (1992), Baublitz and Shimony (1996), Belousek (2003), Solé (2013), Suárez (2015), Hubert and Romano (2017). Under these circumstances, the resulting problem of choice among many Bohmian interpretations has been standardly addressed by appealing to value-laden desiderata, called *theoretical* or *metaphysical virtues*, aiming to be satisfied by a single or a set of these interpretations (e.g., explanatory virtues, ontological parsimony, etc.). However, instead of evaluating the tenability of the Bohmian ontology in terms of its compliance with these theoretical or metaphysical virtues (i.e., the standard way to address the underdetermination issue), an alternative debate has been centered in the context of the choice between different interpretations, each of which associates the fundamental ontology of this theory with a particular mathematical space where it lives —provided this ontology is distinguished from its mathematical representation. More specifically, the contention to be overcome by this alternative debate is between the *three-dimensional worldview*, according to which the fundamental ontology of the theory lives in the three-dimensional space of our everyday experience (Monton, 2006; Allori, 2013; Esfeld, 2014); and the *high-dimensional worldview*, in which it lives in an abstract high-dimensional space (Albert, 1996, 2013, 2015; Ney, 2015).[2]

Considering this observation, the purpose of this contribution is twofold: firstly, to challenge the premise that the problem of choice among the three/high-dimensional Bohmian worldviews can be *solved* by appealing to some theoretical or metaphysical virtues; and secondly, that this problem can be approximately *dissolved* by means of advocating a Bohmian structural realist framework. In so doing, I will proceed as follows: In Section 2, I shall start with a brief theoretical outline of BQM, followed by a description of the three/high-dimensional worldviews compatible with this theory. In Section 3, I shall argue that no proposed set of theoretical or metaphysical virtues, such as *explanatory power*, *ontological parsimony*, *common sense* and *ontological continuity*, can overcome the resulting underdetermination problem. After assessing the possible alternatives to solve or dissolve this problem, in Section 4 I shall suggest a structural realist interpretation through the use of symmetry considerations, according to which the three/high-dimensional distinction is just apparent as both Bohmian views share

[2] Based on Monton (2006) and Albert (2015), I am excluding the most radical and problematic case, mistakenly attributed to Bell (1981, p. 128), in which both spaces are real and fundamental, whilst I am including the case in which both are real but one is fundamental and the other is not.

the same fundamental structure, identified by the symmetry group of BQM. Finally, Section 5 will be left for concluding remarks, followed by an additional section of Appendices.

2. The Bohmian Formulation and Interpretation

Broadly speaking, BQM is a non-relativistic theory of QM that is not physically but empirically equivalent to standard QM. This alternative theory is deterministic and governs the behaviour of a quantum system which, according to the most popular view, is constituted by N objective spinless particles moving in the manifested three-dimensional space $\mathbb{R}^3$ along definite trajectories of the form $\{\mathbf{Q}_i(t)\,|\; i = 1, \cdots, N\}$ —in a similar fashion to the motion of Newtonian particles.

The way each of these particles moves, however, significantly differs from Newtonian mechanics, in the sense that it depends both upon the instantaneous *configuration* of all the particles $\mathbf{Q}(t)$ at time t and on the *wave function*[3] of the composite system $\Psi_t(\mathbf{q},t)$, where $\mathbf{q} = (\mathbf{q}_1,\cdots,\mathbf{q}_{3N})$ are coordinates defined in a high-dimensional space (known as *configuration space*).[4] Also associated with a pilot-wave (presupposing that it guides the particles in some way), the wave function obeys the usual *Schrödinger equation*, whereas the motion of the particles is determined according to the *guiding equation*:

$$\mathbf{V}_i(\mathbf{Q},t) = \frac{\hbar}{m_i}\frac{{\Psi_t}^*\nabla_i\Psi_t}{{\Psi_t}^*\Psi_t}(\mathbf{Q},t)$$

This law of motion is introduced into the theory in addition to the standard QM formulation, where the i-field $\mathbf{V}_i$ corresponds to the three-dimensional velocity of each particle at a given time t, generally defined as the rate of change of position, and $(\mathbf{Q}(t_0),t_0)$ is the initial condition.[5] As

[3] The wave function is a differentiable function $\Psi_t : \mathbb{R}^M \rightarrow \mathbb{C}$ with $\Psi \in L^2(\mathbb{R}^M, \mathbb{C})$ and domain in $(\mathbf{q},t)$. $L^2(\mathbb{R}^M, \mathbb{C})$ is the set of square-integrable functions in M-dimensional configuration space with 1 complex components.

[4] This is a special coordinate ordering of a 3N-dimensional Euclidean space consisting of N three-ordered tuples of coordinates $(\mathbf{Q}_1(t), \mathbf{Q}_2(t), \mathbf{Q}_3(t)), (\mathbf{Q}_4(t), \mathbf{Q}_5(t), \mathbf{Q}_6(t)), \cdots, (\mathbf{Q}_{N-2}(t), \mathbf{Q}_{N-1}(t), \mathbf{Q}_N(t))$, corresponding to each subsystem in three-dimensional Euclidean space. It is called configuration space because we are introducing additional structure into the Euclidean 3N-dimensional space, in such a way that a single point of $\mathbb{R}^{3N}$ represents N points in physical space.

[5] NB, if our focus is not upon predictions but about the fundamental nature of the entities posited by BQM, then the universal function and the configuration of the particles

one can read from this equation, the instantaneous velocity $\mathbf{V}_i$ depends on $\Psi_t(\mathbf{q},t)$ and $\mathbf{Q}(t)$, namely, it is determined by the dynamical evolution of the whole system.[6]

Notwithstanding that this popular formulation of BQM looks rather readable and simple, it does not exhaust all interpretative possibilities. Since we would like to be as general as possible as regards the interpretation of the theory, we cannot only consider this particular presentation of BQM, which suggests that the wave function lives in an abstract high-dimensional space and "guides" a family of particles living in the three-dimensional physical space. Although the particles are essential features of BQM and cannot be removed from the ontology of the theory regardless of where they live (either in the three-dimensional physical space or in a high-dimensional mathematical space), the wave function does not necessarily have to exist as a *beable*: it may determine a set of existing properties indirectly, but does not necessarily have to be regarded as part of the ontology of the theory —understood in the standard way as a 'substance' or object with 'essential' or intrinsic properties. And, even if we believe that the wave function is a *beable*, there are no restrictions as regards being ontologically committed to this object as living in three-dimensional space, as demonstrated by the *multi-field approach* developed by Hubert and Romano (2017). This observation reflects the fact that ontological commitments with respect to any theory do not entail (and have sometimes been mistakenly associated with) the reification and fundamentality of the space in which the theory is mathematically defined. On the contrary, that the theory is *mathematically* defined in an X-dimensional space poses no necessary (in principle) restrictions when conceiving of its underlying *ontology* as living in a Y-dimensional space.[7]

Therefore, we can elaborate two general, incompatible interpretations arising from the BQM formalism: either the worldview according to which

of the entire universe must be considered. As such, $\mathbf{Q}(t)$ describes the configuration of all the particles of the universe at time t and Ψ_t is the function at time t that guides the motion of all these particles taken together.

6 This formulation should consider a series of rules establishing a correspondence between mathematical representations and the empirical consequences of the theory (i.e., the QM predictions). These rules are established by both the *statistical postulate* and a 'measurement framework', described in Bohm (1952b), Dürr et al. (1992) and Holland (1993).

7 It should be noted, though, that a physical theory, as it was originally formulated, already contains an ontology; it is not just a mathematical model. If we recognise this fact, it turns out that the interpretation of such a theory becomes a philosophical, revisionary activity where its mathematical formulation is not necessarily associated with the space in which the ontology lives.

objects and properties live in a real and fundamental high-dimensional space (e.g., *wave function realism*); or the worldview in which objects and properties live in a real and fundamental three-dimensional space, namely, the space of our senses and everyday experience (e.g., the multi-field approach and the *parsimonious approach*, to be defined in the next section).

Having characterised these different interpretations of BQM, let us now address the metaphysical underdetermination that arises as a consequence of trying to advocate a realist commitment with respect to this particular theory —and therefore trying to elucidate a physical worldview compatible with it.

3. The Underdetermination Problem within the Bohmian Theory

There are certainly well-documented cases of underdetermination of theory by evidence where no empirical ground is available to choose one theory instead of another. As we know, this problem may cast doubts on the truth of scientific theories, and hence may be seen as a threat to scientific realism. In response to this problem, some scholars demand that we should engage in inter-theoretic comparisons of relative simplicity, explanatory power, and other non-empirical, theoretical virtues, which may provide rational criteria for theory choice. However, as emphasised by Quine (1975) and numerous contemporary scholars —such as Loewer (1996), De Regt and Dieks (2005), Cohen and Callender (2009), Barrett (2019)— the possibility of engaging in inter-theoretic comparisons based on these virtues may be undermined by at least two reasonable objections. On the one hand, the objection of *immanent comparisons* consists in the claim that these value-laden desiderata are not epistemic but pragmatic and context-dependent, in the sense that they are defined relative to a system of basic kinds and predicates —associated with the linguistic framework in terms of which the theory is framed— that generally differ from one theory to another. The objection of *mutual conflict*, on the other hand, consists in the claim that we cannot compare different theoretical frameworks with relative theoretical virtues because they tend to conflict in a way that there is always a trade-off between them.[8]

Apart from these problems, however, there have been attempts to make comparisons between different postulated or elaborated ontologies, interpretations of mathematical entities, and other metaphysical

[8] One way to avoid this problem might come from providing a metric or score with which theoretical virtues might be weight against each other to arrive at the best balance. However, in order to reach such a balance, the metric balance itself would depend on how we define the theoretical virtues involved inheriting in turn the problem of immanence comparisons (Cohen & Callender, 2009).

commitments in accordance with 'deeper' and more general virtues (i.e., metaphysical virtues) that are not necessarily tied to specific basic kinds and predicates associated with the linguistic framework in terms of which the corresponding theory is framed, such as parsimony, metaphysical explanation and common sense. These virtues have been advocated in response to the stronger case of underdetermination in which there is no scientific ground to decide between different metaphysical commitments compatible with a common body of empirical data *and* other constitutive aspects of a theory. Commonly known as metaphysical (as opposed to theoretic) underdetermination, this case occurs when there are no epistemically accessible features (i.e, both empirical and scientifically) that can distinguish one compatible interpretation from another (van Fraassen, 1991, p. 491). Considering this stronger form of underdetermination, a similar pair of objections can be raised against comparing different metaphysical frameworks with relative virtues of this kind. In this case, the first objection would say that we cannot compare different metaphysical frameworks with relative metaphysical virtues because, although they are not necessarily tied to specific kinds or linguistic predicates, they share the pragmatic and context-dependent nature of theoretical virtues, the only difference being that metaphysical virtues are defined with respect to certain metaphysical commitments that change from one interpretation of a theory to another (Bricker, 2020). The second objection, therefore, would consist in the claim that different metaphysical frameworks with relative metaphysical virtues cannot be compared because a trade-off between them always exists (Bennett, 2009; Kriegel, 2013).

Let us provide some concrete examples based on BQM that support this pair of objections raised against comparing different interpretations in accordance with four metaphysical virtues: common sense, metaphysical explanation, ontological parsimony and ontological continuity.

3.1. Underdetermination: common sense

In the high-dimensional worldview, the set of N three-dimensional particles are not real or are non-fundamental objects at best[9]; they are

[9] According to the high-dimensional interpretation, there are two exclusive views regarding the metaphysical status of the particles: one may either interpret them as non-existing 'shadows' or 'illusions' that arise from an abstract high-dimensional real world or confer upon them an ontological status but of a non-fundamental category compared to that of the fundamental high-dimensional ontology. Perhaps, Valentini (1992) can be read in terms of the first approach, whilst Bohm (1952a,b), Bohm and Hiley (1993) can be read in terms of the second one.

interpreted as N 'shadows' or 'reductive' manifestations of a real and fundamental world particle moving along a curve in a 3N-dimensional Euclidean space. Considering that a prerequisite for any Bohmian realist is that at least a single particle must exist without this implying that any set of particles must live in the three-dimensional physical space, one may depart from the manifest image of the physical realm and imagine a completely unfamiliar world in which there exists a single world particle (as opposed to N particles) whose trajectory is determined by a real field spread throughout a 3N-dimensional Euclidean space (Albert, 1996, 2013, 2015; Ney, 2015). As such, the wave function represents a 3N-dimensional classical field by virtue of the facts that energy and momentum conservation is ensured; and the field represents fundamental intrinsic properties *of* the world particle: it is an assignment of intrinsic properties to each point in a fundamental 3N-dimensional Euclidean space such that the values of these properties at each point in this space is the amplitude of the field at that point (Loewer, 1996).

As a consequence of this characterisation, some scholars —such as Allori (2013), Hubert and Romano (2017)— have argued that this high-dimensional worldview represents a significant departure from our familiar way of interpreting things (what is called *common sense* in the literature), normally associated with the manifest image of the physical, three-dimensional realm. Since the wave function and the world particle are assumed to be living in an unfamiliar high-dimensional space, the ontology posited by this interpretation is not only very abstract, but also is vague with respect to our familiar way of interpreting the referring terms of BQM, such as in the case of the multi-field and/or only the three-dimensional particles.

However, the previous argument has been attacked by some scholars who think that the virtue of common sense is also compatible with a high-dimensional interpretation. Following the analysis of Ney (2015), one can also advance the claim that this high-dimensional interpretation defines a *local ontology* because there is only one world particle whose motion is completely determined by the local assigning values of the field; and furthermore, it also defines a *separable ontology* because it involves an ontology of objects located (or properties instantiated) at distinct regions of a 3N-dimensional Euclidean space in addition to the fact that all categorical features are determined by features of both the field and the world particle instantiated at a point of an individual region in this space. Thus, our familiar concepts of locality and separability, which are foundational aspects of the classical world (excluding gravity, of course), can only be satisfied in a high-dimensional interpretation. And if these classical concepts (as opposed to the manifest image of the physical realm) constitute what we associate with common sense but are not compatible with the three-dimensional ontology, then the three-dimensional worldview can also be conceived as departing from common sense.[10]

It follows from the previous argument that common sense is compatible with one of the three/high-dimensional Bohmian views depending on the sense we give to this metaphysical virtue, either in terms of the dimensionality of the space in which the ontology lives, or in terms of locality and separability. This leads us to argue for the right sense in which common sense must be interpreted, but assuming that the notions of locality and separability are independent from the dimensionality of the space, a question of metaphysical choice between different interpretations turns out to be a question of semantic and pragmatic definition with respect to what is supposed to be common sense to begin with. Therefore, as far as Bohmian interpretations are characterised in terms of the three/high-dimensional space distinction, common sense does not seem to be a reasonable criterion for interpretative choice.

3.2. Underdetermination: explanatory power and ontological parsimony

In the three-dimensional worldview, there are N particles with definite positions and intrinsic properties, such as mass, charge, and so on (i.e., the references of the wave function-independent parameters), whose motion is represented by curves in a real and fundamental three-dimensional Euclidean space, namely, the space of our senses and everyday experience. The dynamics of these particles is specified by the wave function, which evolves according to the Schrödinger equation. However, the fact that these N particles exist does not necessarily imply that the wave function also exists as a beable and represents a physical entity. Initially developed by Dürr et al. (1992, 1995) and Goldstein and Zanghì (2013), the *parsimonious approach* interprets the wave function not as an object or property; it is rather a mathematical, nomological entity that determines,

[10] One might say that Ney's high-dimensional locality is not what we commonly have in mind when speaking about locality. However, if we differentiate Ney's high-dimensional locality from its three-dimensional counterpart, it is precisely the assumption that the ontology lives in a three-dimensional space (as opposed to its local nature) which is doing the job of choosing the correct interpretation on the basis of common sense. Thus, our claim here has to do with interpreting locality as a self-standing criterion associated with common sense, independent of the space in which this notion is defined. Once we associate this notion with such a space, the criterion used for interpretative choice does not correspond to the notion of locality anymore. Thanks to an anonymous reviewer for encouraging me to explain this point.

describes or prescribes (depending on our preferred view of laws of Nature) the dynamics of the particles living in the three-dimensional physical space. Moreover, according to this interpretation, we say that BQM is explicitly non-local by virtue of the fact that the dynamical properties of any particle, its velocity in particular (i.e., the reference of the field defined above), not only depend on its position, but also on the positions of the rest of the particles and the wave function. This non-local behavior of the particles, together with the nomological status of the wave function, implies that the only real properties that are intrinsic to the constitution of the particles are not their dynamical properties (e.g., energy, momentum, etc.), but their positions extended along trajectories —and the wave function-independent parameters such as mass, charge, and so on.

As a consequence of this characterisation, some advocates of the parsimonious approach, such as Goldstein and Zanghì (2013), have argued that the three-dimensional worldview represents the most parsimonious alternative compared to any other interpretation. Moreover, they have also argued that the mere fact that this approach interprets the wave function as nomological implies that the non-classical effects acquired by the particles cannot be explained in terms of any causal interaction with other objects. In other words, since the motion of the particles cannot be explained by any existing causal-mechanical agent, non-locality remains an unexplained feature of the theory —relative to the causal-mechanical account of explanation.[11] Therefore, we end up with a trade-off between parsimony and explanatory power.

However, although these arguments have been the dominant, prevailing view with respect to BQM, one may also argue that the three-dimensional Bohmian world is compatible with the alternative view, proposed by Hubert and Romano (2017), that the wave function represents a non-local and non-separable multi-field living in the three-dimensional space.[12] The nature of this multi-field can be defined in terms of the facts that energy and momentum conservation are ensured; and instead of being

[11] As corroborated by the problem of immanent comparisons, this conclusion depends on the causal-mechanical account of explanation and does not necessarily follow from taking an alternative account compatible with this parsimonious view. It is precisely this point that enables to argue that explanation is a pragmatic, immanent and contextual metaphysical virtue. In particular, it is immanent and contextual in the sense that it is defined relative to a causal-mechanical account. It is partly because of this immanent and contextual nature, that this and other metaphysical virtues cannot be used as criteria for interpretative choice, as it is concluded in this manuscript.

[12] This idea has not only been suggested in Hubert and Romano (2017), but also in Norsen et al. (2015). However, I consider the former to be the most comprehensive interpretation to date among this kind.

an assignment of a definite value of intrinsic properties to each point of three-dimensional space (as in the case of a classical field), it is rather a more general assignment of a precise value of properties (not intrinsic in general) for an entire N-tuple of points of three-dimensional space. Moreover, the dynamical non-local correlations are causally explained in terms of an exchange of energy between the Bohmian particles. Such a mechanical process is ensured by the fact that the value of the multi-field is instantaneously specified for N-tuple of points in three-dimensional space. In this sense, the non-local character of the theory is characterised in terms of the nature of this holistic or relational multi-field, rather than in terms of the 'obscure' notion of *action at a distance* taking place between the particles. This, of course, makes the theory explanatory superior to the last alternative —relative, again, to the causal-mechanical account of explanation. However, the price to be paid is that this interpretation turns out to be ontologically robust in the sense that it introduces the multi-field as part of its ontology. In this respect, apart from the trade-off between explanatory power and ontological parsimony in this context, the three-dimensional worldview does not commit us to one of these metaphysical virtues, it is rather compatible with both.

Analogously, Goldstein and Zanghì (2013) have critically argued that the high-dimensional worldview is the most ontologically robust alternative compared to any other interpretation due to the fact that the ontology of BQM is given in terms of a world particle and a field living in a high-dimensional space. In this case, the non-classical effects acquired by the particles in three-dimensional space are just 'illusive' or 'reductive' manifestations of a local and separable ontology living in a high-dimensional space. So, there are no non-local effects to be explained after all. However, the tenability of this dominant view can be as well challenged. We can advance the claim that the high-dimensional worldview is also compatible with the view that the wave function is not a real object but is interpreted as a law of Nature. Thus, it is an interpretation more parsimonious than wave function realism in the sense that it does not introduce any field as part of its ontology. However, the price to be paid for those who prefer 'desert landscapes' is that this interpretation turns out to be less explanatory to its high-dimensional alternative not because the wave function cannot causally explain the non-local behavior of the hypothetical particles in three-dimensional space, but because it cannot causally explain the motion of the actual world particle, whatever the motion of this entity might be.[13] Therefore, we end up with

[13] One possible virtue associated with this interpretation is the fact that, for a many-body system of N particles, the world particle describes trajectories that never cross,

a similar conclusion: apart from the trade-off between explanatory power and ontological parsimony, the high-dimensional Bohmian world does not commit us to one of these metaphysical virtues, it is rather compatible with both.

Under these circumstances, since both worldviews do not have a preference towards one of the above virtues, apart from the fact that these virtues are always trading-off between them, there is no way to choose one of these interpretations based on them. It follows that these virtues cannot be the most reasonable criteria for interpretative choice.

3.3. Underdetermination: ontological continuity

Let us finally investigate another criterion that might be conceived as a metaphysical virtue for interpretative choice between the three and the high-dimensional worldviews without falling into the last two objections. This criterion, called *ontological continuity*, relies on the assumption that the preservation of all and only the theoretical referring terms of successful physical theories that are indispensable for obtaining the predictions of the relevant phenomena (normally called 'working posits') is a reliable guide to approximate truth. We can take this criterion to mean that, other things equal, if one interpretation of a successor theory approximately preserves the working posits of a predecessor theory, then we ought to favour that interpretation of the successor theory (in the sense that it is reasonable to believe that it is approximately true).

Note that this way of defining ontological continuity is immune to the problem of immanent comparisons by virtue of the fact that the definition itself is not contextual and can be applied to any set of successive theories, irrespective of the kind of ontology and the metaphysical commitments underlying these theories. In the same way, it is less (although still) probable that there is a trade-off with other metaphysical virtues because it is not associated with only a single theory but with two or more successive theories, something which broadens the scope of compatible interpretations that are virtuous in many other ways.

For some scholars, such as Allori (2017), Esfeld and Deckert (2017), the three-dimensional worldview is the only metaphysical framework capable of satisfying the previous metaphysical virtue as regards the classical-quantum transition. They argue that a Bohmian extension to the classical domain can only be articulated by interpreting the Bohmian

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whilst three-dimensional particles may in principle cross. This might avoid certain interpretative problems associated with BQM.

particles and their positions (as opposed to the wave function and its phase) as preserved working posits by virtue of the fact that such an ontology is indispensable for obtaining the empirical predictions of both domains, and is, to some extent, preserved during this theory shift (ignoring the details about their actual nature). However, as I have recently argued in Manero (2024), the previous argument is objectionable because the classical analogs of the wave function and its phase can also be interpreted as preserved working posits, provided a notion of indispensability is expanded to incorporate successful explanations (in addition to predictions) of the relevant phenomena.[14] According to Manero (2024), the phase of the wave function is an approximately preserved working posit that can also be interpreted as a high-dimensional field —following an interesting analogy with optical phenomena via the Hamilton-Jacobi formulation. Thus, all things being equal, both the three-dimensional particles ontology and the high-dimensional wave function phase may be regarded as approximately preserved working posits which form the basis of an equally reliable endorsement to the above metaphysical virtue in a sufficiently virtuous way. Therefore, if the author's continuity argument is right, there are fairly reasons to conclude that the metaphysical underdetermination between a three and a high-dimensional worldview cannot be broken in general via ontological continuity.

4. Dissolving the Underdetermination Problem

As far as the three/high-dimensional distinction is considered, we end up with the presence of a strong metaphysical underdetermination by theory that cannot be broken by appealing to well-known metaphysical virtues, such as common sense, explanatory power, not less to say, ontological parsimony and ontological continuity. This implies that no matter if the best arguments are brought out to endorse one of the horns of the alleged underdetermination, the conclusion is that these virtues are not sufficient to decide which Bohmian view is the correct one. At the end of the day, this conclusion reinforces some well-known arguments that

[14] Considering this broader indispensability criterion, the classical analogs of the wave function and its phase can also be interpreted as preserved working posits. This is because, although the phase is not indispensable for deriving the predictions of classical mechanics, it is indispensable for explaining the phenomena under the lens of the Hamilton-Jacobi framework in the broader context of classical and quantum mechanics. Indeed, without positing the phase as part of the ontology of both theories, we could not explain the motion of the corresponding world particles through a clear story of how the physical world actually is according to these theories.

reveal the limitations of theoretical and metaphysical virtues as criteria for interpretative choice: that non-empirical virtues, such as common sense, explanatory power, ontological parsimony and ontological continuity, are not capable of providing a uniquely determined and fully objective choice for all contexts (Acuña & Dieks, 2014).

In view of the previous discussion, we end up with two options on the table. On the one hand, one might reasonably embrace the problem and accept the underdetermination at the expense of advocating a pragmatist stance that basically would undermine the (metaphysical) realist basis that makes the alleged underdetermination an actual problem. On the other hand, one might revise and undermine some of our deeply rooted standard metaphysical presuppositions (apart from the metaphysical commitments and value-laden assumptions already considered) that, to a certain extent, are responsible for originating the underdetermination problem. More specifically, the latter option dissolves this problem because it reveals that the metaphysical underdetermination, as it is originally formulated in the context of BQM, is only compatible with the standard metaphysics of individual objects, namely, the view that objects are the fundamental bearers of a bundle of instantiated intrinsic properties that individuate them. Let us analyse both options in more detail.

4.1. Endorsing object-oriented metaphysics

As we know from influential literature, accepting the underdetermination problem leads to a fatal flaw against *metaphysical realism*: the thesis that, apart from the existence and independence assumptions associated with any kind of realism, endorse the epistemological claim that there is a one-to-one truth-correspondence between our representations (e.g., our preferred Bohmian interpretation) and the external world. Making an explicit departure from this way of understanding realism, one might reasonably suggest a realist stance along classic-pragmatist lines that does not collapse into a skeptic or constructive empiricist stance towards metaphysical underdetermination, nor does it departs from a naturalistic conception of metaphysics informed by current science to reach the speculative arena of the old rationalist systems of thought.

According to Howard (2011), one might plausibly argue that the most appropriate epistemic stance with respect to scientific theories or their interpretations is neither belief nor mere acceptance but a kind of Peircean 'pursuitworthiness'. This stance would allow one to accept the resulting underdetermination at the current stage of knowledge by collecting an inventory of possible Bohmian interpretations that exhibit some theoretical and metaphysical virtues, irrespective of whether or not they are objectively true. As French (2014, p. 42) comments on Howard, this inventory might enrich (partially and progressively) our understanding of the world, in a way that it involves (at least) a tentative and preliminary attitude toward our best scientific theories. This would mean that our accessibility to the world is constrained by our current empirical and epistemic limitations, allowing us to acquire *modal understanding*, in the sense of having different perspectives of the actual world according to our best, current scientific theories.

A similar view has been recently advocated by Barrett (2019). At the end of his contribution, he explicitly says: "With respect to the proper choice of empirical ontology and debates concerning such things as primitive ontology and configuration space realism more specifically, there is arguably not much to be gained by trying to stipulate an intuitively preferred metaphysics up front (Barrett, 2019, p. 232)." He then recapitulates his pragmatist view on the matter saying that "given our epistemic situation, one might more profitably adopt a flexible view of the matter without trying to stipulate how our experience ought to supervene on the physical world once and for all (Barrett, 2019, p 232)." On these passages, Barrett is basically revealing the pragmatist role of metaphysical explanation in science due to the prerequisite of our best scientific theories to account for their empirical adequacy in a reasonable explanatory basis (i.e., among other things, being explicit with respect to presuppositions concerned with our accessibility to experience that normally remain unnoticed). Indeed, his thesis can be read in a way that makes the first option amendable against anyone intending to find a single and compelling account of experience through our best scientific theories.

4.2. Rejecting object-oriented metaphysics: ontic structural realism

The other option is now on the table. Why do not we simply undermine the object-oriented metaphysical presupposition according to which the postulated ontology, living either in a three or in a high-dimensional space, is constituted by fundamental, distinguishable individual objects as bearers of a bundle of intrinsic physical properties?

Based on the stipulation that the metaphysical category of relations and structures is ontologically prior to the category of individual objects, the claim is to put forward a view of the world supported by BQM according to which the self-standing relations and structures represented by the relevant equations and Bohmian laws (i.e., the guiding and the Schrödinger equations) form the fundamental (or the only existing) categories of the world over and above the individual objects and properties originally posited by this theory (i.e., the relata which stand among these relations and structures). The project known as *ontic structural realism* applied to the Bohmian context is that of developing this claim rigorously (Ladyman et al., 2007; French, 2014).[15] Although this project is primarily motivated by providing a theoretical ground for structural continuity in scientific change (in the general context of the so-called 'pessimistic induction argument'), we shall only endorse this realist thesis based on certain metaphysical inferences obtained from BQM: the ontological priority of structures and relations over and above other categories or aspects of the word, provided these relations and structures are epistemically accessible by means of mathematical representations, such as the physical laws and other mathematical structures underlying this theory.

Under these circumstances, the metaphysical underdetermination between different object-oriented Bohmian interpretations can be dissolved by virtue of the fact that the ontologies posited by these interpretations can be conceived as different descriptions of the fundamental structure of the Bohmian world. More specifically, if the fundamental ontology of BQM lies at the level of the structure (i.e., represented by the laws of this theory), then we can conceive of the three/high-dimensional views as two different ways to describe the fundamental ontology of BQM. The underlying reason is that, according to ontic structural realism, the Bohmian particles, together with the field, the multi-field, and the world particle, are not objects in the standard metaphysical sense (i.e., individual bearers of intrinsic properties). Rather, they are either physical modes or features of the fundamental structure of the Bohmian laws (in the case we endorse the eliminativist version of ontic structural realism); or non-fundamental objects whose properties are relational in the sense that are ontologically dependent upon such a structure (in the case we endorse a non-eliminativist version). Contrary to Quine's criterion of ontological commitment, as defined in Quine (1948), we can advance this argument in more formal terms by advocating, for example, the truth-maker theory of Cameron (2008), and conceive of the three/high-dimensional views as

[15] Note that there are many versions of ontic structural realism. The eliminative thesis associated with this project is the most radical structural position and consists in the claim that all things in the world constitute a nexus of fundamental relations and structures with no recourse to the existence of the relata upon which these relations and structures take place. However, ontic structural realism is also compatible with a view that does not eliminate objects. Thus, one might be inclined to think that relations and structures are fundamental, in the sense that they are ontologically prior to existing relata.

two different ways to make sense of the theory through the elucidation of different true basic predicates, meanwhile the fundamental relations and structures represented by the Bohmian laws are the truth-makers of these basic predicates.

4.2.1. Objections against ontic structural realism in the Bohmian context

Unfortunately, at least two possible objections may be posed against this strategy (considering only relevant objections to the subject matter of this contribution). Firstly, that (i) the system of Bohmian laws are expressed in terms of object-oriented language, and that it is almost impossible to prescind of this language and try to isolate a core set of purely relational mathematical structures that can explain alone the empirical predictions of BQM; and secondly, that (ii) these laws are defined in configuration space, which is intrinsically defined in a three-dimensional space structure, hence favouring the three-dimensional view. For sake of simplicity, let us first address the first objection, and then proceed to address the second one. As we shall see, the response to this second objection requires certain mathematical concepts that, to the best of my ability, I shall try to explain in more simple terms, relegating the technical details to the Appendices A and B.

4.2.2. No problem with object-oriented language

One of the advantages of abandoning Quine's criterion of ontological commitment is that we do not need to reduce ontological questions to linguistic facts. On the contrary, as ontic structuralists we believe that basic kinds are not necessarily tied to the values of the variables that lie within the domain of the quantifiers —if the relevant sentences of the theory are to be held as true—, but are just those fundamental constituent entities (e.g., extensional relations between variables) that have to exist in order to make the relevant sentences of the theory true.

Thus, although any set of physical laws and principles is expressed in terms of both natural and mathematical languages, we do not need to detach the object-oriented semantics from the syntactic structure of these languages (i.e., we do not need to regiment the theory in a special way), because we can allow a theory to assert true sentences about objects without these objects being ontological commitments of that theory. This observation would allow us to present the underlying fundamental structure of the Bohmian world (in terms of which physical laws and principles are framed) by means of certain relatively-structured mathematical formulation without having to demonstrate —via a purely-structural, philosophical representation— that this formulation is strictly detached from any object-oriented semantics.

However, since the ontological commitment with respect to BQM is at the structural level, regardless of its linguistic representation, we still need to account for the empirical adequacy of the relevant structural ontology, even if this criterion is assumed to be satisfied at the object-oriented level. More specifically, we need to explain how both the manifested macroscopic world and the phenomena successfully predicted by BQM, ought to depend (or emerge), metaphysically speaking, on (from) the fundamental structural ontology. Although I will not address this issue here (i.e., the structural counterpart of what is known as the *macro-object problem* in the literature), there is a contention between those that believe that this problem cannot be overcome (Maudlin, 2019, Ch. 4), and those that have articulated certain metaphysical strategies to provide a satisfactory answer (French, 2014, Ch. 7). Let us proceed to address the second objection (ii).

4.2.3. Against a preferred space dimension

As already mentioned, BQM is mathematically defined in configuration space. As a result, the description of a N-particle system by this theory fixes a natural coordinate-ordering of an arbitrary 3N-dimensional Euclidean space in terms of N three-ordered tuples of coordinates ($\mathbf{Q}_1(t)$, $\mathbf{Q}_2(t)$, $\mathbf{Q}_3(t)$), ($\mathbf{Q}_4(t)$, $\mathbf{Q}_5(t)$, $\mathbf{Q}_6(t)$), $\cdots$, ($\mathbf{Q}_{N\text{-}2}(t)$, $\mathbf{Q}_{N\text{-}1}(t)$, $\mathbf{Q}_N(t)$), corresponding to each subsystem in three-dimensional Euclidean space. This natural ordering compels us to write the laws of the theory in the three-dimensional space. In particular, the guiding equation is written in terms of a set of N differential equations, each one defined in a three-dimensional vector space. This means that a single parametrised curve in configuration space uniquely defines N parametrised curves in three-dimensional space.

Considering this observation, one might argue that any ontological commitment with respect to the mathematical structure of the laws of the theory, provided this structure is regarded as ontologically prior to other non-structural categories, is implicitly favouring the three-dimensional view: the fundamental space of the theory seems to be the three-dimensional physical space by virtue of the fact that the fundamental structural ontology (i.e., the Bohmian laws) lives within this space. This would beg the question as regards the choice between both three/high-dimensional worldviews.

Fortunately, this objection can be reasonably overcome without renouncing to the project of ontic structural realism. In so doing, one should note that it is impossible to introduce a natural ordering into a given Euclidean structure without appealing to any additional, external element. This means that we cannot construct the configuration space out of an arbitrary 3N-dimensional Euclidean space without introducing by hand the natural ordering of the three-dimensional space. Granted this, one might incorporate, in addition to the 3N-dimensional Euclidean space, a fundamental dynamical structure that induces such a natural ordering. However, in order to avoid the force of the last objection, this dynamical structure should be one which is ontologically prior to any other Euclidean structure and which is not mathematically defined neither in the three-dimensional space nor in the 3N-dimensional Euclidean space. Otherwise, one would be begging the initial question by introducing by hand what is in principle required to decide between the two rival Bohmian worldviews.

Thus, my suggestion is that this fundamental dynamical structure should be dimensionless in the Euclidean sense. In other words, this structure should have the property of being defined in a mathematical space which is not Euclidean and whose dimension is not Euclidean. Considering this suggestion, we would have an Euclidean-dimensionless dynamical structure at the top fundamental level of reality, and the three-dimensional structure of the laws of the theory would arise from this fundamental structure without having any privilege role to play within it. An important question still remains: *Is there such a fundamental dynamical structure?*

To answer this question in the positive, let us first endorse *group structural realism*, a particular way of interpreting ontic structural realism. Along the lines of Brian Roberts, we can put forward the claim that "The existing entities described by quantum theory [BQM in this case] are organised into a hierarchy, in which a particular symmetry group occupies the top, most fundamental position (Roberts, 2011, p. 50)." Granted this claim, we can proceed to demonstrate that the symmetry group of BQM can be associated with a dimensionless fundamental dynamical structure from which the non-fundamental three-dimensional profile of the Bohmian laws can be induced in a natural and structural way. Let us corroborate the reliability of this suggestion by completing the following three-fold tasks: firstly, we start by identifying the mathematical language in terms of which symmetry groups are framed; secondly, we briefly reveal the relatively-structured nature of this mathematical language; and thirdly, we proceed to identify the symmetry group of BQM and demonstrate that this mathematical structure is dimensionless in the Euclidean sense.

As argued by French (2014, Ch. 5), *Lie group theory* is one of the most suitable mathematical languages in terms of which the fundamental dynamical structures underlying our best scientific theories can be presented. This is partly explained by virtue of the fact that the expressive power of Lie group theory has been useful in elaborating new formulations or reformulations of many successful physical theories capable of generating empirical predictions and of improving our understanding about them (e.g., their inter-theoretic relations, their mathematical foundations, and their physical limitations and possibilities.). In order to understand how Lie group theory has become physically relevant and theoretically successful (in the sense just described), we should recapitulate the central technical aspects that makes this mathematical language work in the appropriate way.

Simply put, a *group* G consists in a set of (finite or infinite) elements with an operation defined between two of these elements to form a third element within the group by means of a *multiplication rule* $z = x * y$, where $x, y, z \in G$. The set and the operation must satisfy the associative property and must have both an identity element and an inverse element. If the group is a differentiable manifold and the operation $*$ is smooth (i.e., it defines a continuous mapping with derivatives of all orders) then it is called a *Lie group*. As one can easily note, this abstract definition does not entail a straightforward physical interpretation. However, this interpretation can be elaborated by means of the following technical observations:

There are geometrical objects upon which a Lie group acts. These can be figures composed of spatial points, such as triangles or squares, but they can also be vectors, differential forms, tensors, etc. The action of a Lie group on a set of these objects (e.g., a vector space, an affine space, etc.) is a 'copy' of the Lie group defined in that set, in the same way a self-portrait is a copy of the artist represented in a canvas. In the particular case of a Lie group G acting on a vector space V (a case which will be our main focus in this paper), the action of G on V is, in formal terms, a *Lie representation* of G in V: a function that goes from G to the group of automorphisms of V.[16] Since these automorphisms are realised as a set of linear transformations or mappings between initial and final points in V, it follows that any element of an abstract Lie group G can be interpreted as (or induces via a Lie representation) linear transformations in V. For example, the set of rotations in three-dimensional space are linear transformations induced by the Lie representation of the set of 3×3 orthogonal matrices with determinant equal to one, called the *special orthogonal group* SO(3), on $\mathbb{R}^3$. Thus, in general we have two conceptually different mathematical objects that are equivalent (up to a Lie representation): the abstract Lie groups, on the one hand, and the induced transformations in the mathematical space

[16] The state space of a general theory T is not in general a vector space (e.g., the affine Minkowski space of special relativity). However, since this contribution is about non-relativistic QM, only vector spaces are required.

in which these Lie groups are Lie-represented, on the other. Granted this equivalence relation, let us figure out the connection between the abstract group-theoretic domain and the physical domain.

As discovered by Weyl (1931) (through what is known as Weyl's programme), Lie groups are abstract mathematical objects that have no physical interpretation unless they act upon the state space V of a physical theory T (hereinafter, vector space V of T). As demonstrated by him, the way this interpretation is revealed is through the concept of invariance: the Lie representation of a Lie group G in V takes the form of linear transformations that leave certain sorts of objects invariant, which are interpreted as physical symmetries. In particular, the induced transformations of G can be associated with the dynamical evolution of a physical system described by the laws of T.

Considering this association, we say that the abstract Lie group G is a *symmetry group* of a theory T describing some physical system if the fundamental laws of T are invariant under the induced continuous transformations of G in the vector space V of T (i.e., the Lie representations of G in V). Since the laws of a theory T can be reformulated in this way —as invariants of a certain symmetry group G—, the identification of G (together with the vector space V of T in which G is Lie-represented) is sufficient to determine (up to approximations and idealisations) its dynamical structure, and furthermore, the complete laws of T can be obtained by Lie-representing G in V. It is partly due to this illuminating bridge between abstract mathematics and dynamics in physics that makes group theory both predictably and theoretically successful. Once we have introduced the central technical elements of Lie group theory, let us proceed to address our second task.

Based on the fact that any Lie group element induces a group transformation on a vector space, we can interpret any Lie group as an abstract mathematical object that generates (physical) binary relations on the vector space in which it is Lie-represented. For example, any simple rotation in the three-dimensional Euclidean space, induced by any element of the SO(3) acting on that space, can be interpreted as a binary relation between the initial and the final coordinates of the rotation angle. This alone suffices to convince us that any Lie group is an abstract mathematical object that induces a physical interpretation intrinsically relational or structural when they are Lie-represented in vector spaces.

But we can advance this claim as regards the relational structure of Lie groups themselves without being Lie-represented in vector spaces. According to Arthur Eddington, the Lie group language expresses (in abstract terms) the second-order relationships that hold between physical relations. In his own words, "Whatever the nature of the entities, the use of group theory allows us to abstract away the 'pattern' or structure of relations between them. What the group-structure represents, then, is the 'pattern of interweaving' or 'interrelatedness of relations' (Eddington, 1941, pp. 137-140)." This can be illustrated by appealing to the multiplicative law of SO(3), where any element of this Lie group can be obtained by the composition of two or more elements of the same group, inducing successive and continuous rotations in the three-dimensional Euclidean space. Therefore, group-theoretic structures underlying (or compatible with) our best scientific theories are likely to be relational or structural in this second-order sense, completing our second task. Let us now address our third and final task: *What is the symmetry group of BQM?*

The precise answer to this question shall be relegated to the Appendices A and B, where technical concepts and definitions are explained. As for the present purposes, it suffices to make the following important observations, followed by the explicit disclosure of this symmetry group.

Let us recall that the world particle dynamics in BQM is mathematically defined in the 3N-dimensional configuration space. However, as explained in the Appendix A, one can also define such Bohmian particle dynamics in the cotangent bundle of the M-dimensional Euclidean space (hereinafter, 2M-phase space) —in an analogous fashion to the position-momentum phase space in Hamiltonian mechanics—, provided the M-dimensional configuration space of this theory is obtained from the natural ordering of an arbitrary M-dimensional Euclidean space, where M=3N for a system of N particles. Considering that in BQM the physical state of a N-particle system is not only mathematically defined in the 3N-Hilbert space (i.e., the infinite mathematical space of wave functions evaluated in the 3N-dimensional configuration space), but also in the 6N-phase space (i.e., the mathematical space of the world particle dynamics), the symmetry group of BQM should satisfy the following condition: both the Hilbert space and the phase space must be associated with Lie representation spaces in which this symmetry group is Lie-represented.[17]

Granted this condition, my suggestion is to identify the symmetry group of BQM with the *inhomogeneous metaplectic group* IMp(M), which

[17] Note that 3N is not the dimension of the 3N-Hilbert space, but the dimension of the configuration space in which any element of $L^2(\mathbb{R}^M,\mathbb{C})$ is evaluated. In general, the 3N-Hilbert space is infinite-dimensional as wave functions are generally defined as functions of continuous variables. This implies that IMp(M) does not admit Lie-representations of finite dimension; they are in fact unitary representations. Fortunately, we shall see below that IMp(M) can be expressed in terms of finite-dimensional matrices via an algebraic and topological mapping.

is equivalent (up to an algebraic and topological mapping known as *maximal central extension*) to the dynamical group-theoretic structure of Hamiltonian mechanics, i.e., the *inhomogeneous symplectic group* ISp(M) (see definitions in the Appendices A and B).

This identification not only reveals that, from the group-theoretic point of view, BQM can be interpreted as a Hamiltonian theory up to an algebraic-topological mapping, but also that the physical state and the evolution of a Bohmian system, including the Schrödinger equation and the guiding equation, can be completely determined by the Lie representation of IMp(M) in the 3N-Hilbert Space and the 6N-phase space, respectively. In Maurice de Gosson's words:

> [...] Both classical and quantum mechanics rely on the same mathematical object, the Hamiltonian flow, viewed as an abstract group. If one makes that group act on points in phase space, via its symplectic Lie representation, one obtains Hamiltonian mechanics. If one makes it act on functions, via the metaplectic representation, one obtains quantum mechanics. It is remarkable that in both cases, we have an associated theory of motion: in the symplectic representation, that motion is governed by Hamilton's equations. In the metaplectic representation, it is governed by Bohm's equations (de Gosson, 2001, p 267).

Considering that both Hamiltonian mechanics and BQM can be formulated in the same way at the level of an abstract, group-theoretic structure, we may be ready to come back to our original task: *Why the symmetry group of BQM is dimensionless in the Euclidean sense?*

At first sight, there seems to be no difficulties in demonstrating that Lie group theory is a dimensionless mathematical language in the sense the vector space V of the theory defines its dimension (the 3N-Hilbert space and the 6N-phase space in the Bohmian case). The group dimension is defined in terms of the number of independent elements a given Lie group G contains or, more formally, the dimension of its underlying differential manifold. This definition is so abstract that there seems to be no direct connection to the dimension of the vector space V in which G is Lie-represented, formally defined as the number of vectors that the linear basis of V possesses or, in more familiar terms, the minimum number of independent coordinates (degrees of freedom) needed to specify a point within V. However, when we try to interpret the group dimension physically, we can start to see some slight connections with the dimension of V. The question that interests us is whether these connections are such that the symmetry group of BQM (i.e., IMp(M)) can be considered the fundamental dynamical structure from which the Bohmian laws arise as invariants without the three-dimensionality of these laws having a privilege role to play within IMp(M).

Let us remind that each element of a Lie group G induces a transformation in the vector space V where it is Lie-represented. Since G might have different group elements, one also might have different type of transformations in V. For example, the usual Galilean group $G_{gal}(3)$ possesses ten different elements, each one which can be Lie-represented in the Euclidean space as three rotational parameters, three boosts parameters, three space translation parameters and one time translation parameter. For this reason, $G_{gal}(3)$ is an abstract Lie group of dimension equal to ten, whilst its physical interpretation consists in ten different types of transformations occurring in the (3+1)-dimensional Euclidean space. However, it is not the same to have rotations, boosts and space translations in the three-dimensional Euclidean space rather than in the Euclidean plane. If we had two different Euclidean spaces with different dimension (e.g., one of two and another of three, excluding the temporal dimension), the structure of the inducing transformations would differ in the same way that the structure of $G_{gal}(3)$ itself. Although the multiplication law of $G_{gal}(3)$ remains intact, the group dimension differs depending on whether it is Lie-represented in the two or in the three-dimensional Euclidean space, corresponding to six and ten group elements, respectively. Indeed, for any dimension M of the Euclidean space where the general Galilean group $G_{gal}(M)$ is Lie-represented, the group dimension of this Lie group is equal to M(M+3)/2+1. As for a general finite-dimensional Lie group G, this means that, although the second-order structure of G (given by the multiplication law) remains intact, the dimension of G depends on the space in which it is Lie-represented.

Coming back to our main concern, the last conclusion is corroborated by the fact that IMp(M) has dimension equal to M(2M+3)+1, for any M-Hilbert space and for any 2M-dimensional phase space. Thus, we may conclude that, although it is true that the group dimension of IMp(M) is not defined in the same manner as that of the Hilbert space and the phase space, the Lie group structure of as a whole, where the dimension of IMp(M) is included, contains information about the dimension of these Lie representation spaces. This does not mean that IMp(M) possesses alone a three-dimensional structure. Rather, since the multiplication law of IMp(M) is unique and compatible with many group dimensions associated with the Lie representation space, the three-dimensionality of the Bohmian laws arises from IMp(M) by singling out only one particular dimension (i.e., the dimension associated with configuration space M=3N), without IMp(M) being necessarily dependent on a three-dimensional structure.

In this way, it makes sense to argue that the symmetry group of BQM is ontologically prior to the dimension of the Hilbert space and the phase space in which this group is Lie-represented. What is needed, corresponding to our initial concern, is just the stipulation that IMp(M) is the fundamental structure of BQM, from which everything else metaphysically depend. In particular, the dimension M of the Euclidean space associated with the M-Hilbert space and the 2M-phase space arises from this group-theoretic structure as a free parameter that takes any possible integer number, including the one which leads to the Bohmian laws (i.e., when M=3N holds). Let us make four final remarks.[18]

Firstly, considering that for each M the abstract group IMp(M) allows the possibility of having a different induced structure at the level of the Hilbert space and the phase space, one might think that if IMp(M) is the fundamental structure from which everything else metaphysically depend, it has to explain why Bohmian laws are three-dimensional and why we observe macroscopic objects in the three-dimensional physical space. In other words, IMp(M) has to explain alone why the manifested space of our senses and experiences possesses a particular and privileged dimension M=3N without begging the question and recurring to any three-dimensional structure. This is the macro-object problem that was mentioned earlier and, although I recognise that a complete answer to our initial concern cannot be addressed without solving this problem, we need to relegate this issue to another contribution. In other words, the purpose of this contribution is to focus on the underdetermination problem within the Bohmian context, provided that the macro-object problem is assumed to be solved.

Secondly, note that there could be many ways to reformulate the Bohmian laws or any structural Bohmian ontology in a way that we end up with a problem of structural (as opposed to object-oriented) underdetermination within BQM. In other words, one might be skeptic of endorsing group structural realism because nothing ensures us that IMp(M) is the true group-theoretic structure of this theory. For example, this objection to group structural realism is reinforced by Roberts (2011), where it is claimed that there could be an infinite tower of group-theoretic structures homeomorphic to each Lie symmetry group. This argument alone would suffice to argue that if we endorse group-structural realism by making, in particular, an ontological commitment with respect to IMp(M), we end up with a strong and unavoidable structural underdetermination that would undermine the possibility of overcoming our initial concern

[18] Thanks to one of the reviewers for pushing me to emphasize and clarify the third and fourth remark.

regarding the underdetermination between the three and the high-dimensional worldviews. Although I think that this issue cannot be strictly solved as the very nature of metaphysics is to be condemned by its unavoidable condition of being underdetermined by any kind of knowledge, one might propose to see this structural underdetermination within BQM as a question of degree without abandoning scientific realism. That is, we should look at group structural realism as the metaphysical framework compatible with BQM that is relatively less underdetermined than the available alternatives. In more precise terms, the idea is that the more one approaches the fundamental level in the metaphysical hierarchy of the Bohmian world the less underdetermined is the ontology of BQM. As illustrated by the previous analysis, the resulting underdetermination at the non-fundamental level of the object-oriented Bohmian ontology (i.e., the three/high-dimensional worldviews) is relatively maximal, whilst it is relatively minimal at the level of the fundamental structural ontology of the theory (i.e., group-theoretic structures, such as IMp(M)).

Thirdly, related to the previous remark, one might say that, based on the Bohmian formulation expressed in Section 2, the structure underlying BQM is more robust than that of standard QM because it introduces the guiding equation in addition to the standard formulation. Considering this observation, one might wonder whether there are sufficient reasons to claim that, if both theories are empirically equivalent, we should prefer the most robust alternative. However, it should be noted that one cannot decide between the structure of BQM or that of standard QM by means of the metaphysical virtue of parsimony (i.e., the opposite virtue of robustness). As elucidated in Section 3, parsimony is an immanent and contextual criterion. Defined as such, it is not related to the question of truth and cannot solve or dissolve the underdetermination issue. For example, BQM might be a more robust interpretation but at the same time it might be more amendable to ontological continuities, such as in the classical-quantum transition. Furthermore, one of the reasons to prefer a structuralist metaphysics instead of an object-oriented metaphysics is that the former and not the latter may partially respond to the pessimistic meta-induction argument associated with the problem of theory change. The theoretical structure capable of solving this problem should be such that it must exhibit some ‘plasticity’, in the sense that it should be preserved among theoretical transitions. Under these circumstances, BQM’s structure should be preferred due to the fact that it retains its structural content in the classical-quantum transition, as opposed to standard QM’s structure.

Fourthly, it is claimed that BQM remains relevant and continues to be debated because it employs an object-oriented metaphysics, in which the objects are point particles, making it easier to understand and more appealing, as it resembles classical mechanics. However, if one abandons object-oriented metaphysics and adopts group structural realism, where the ontology is more abstract, what justification remains for BQM as a whole, given that it is empirically equivalent to standard QM and has an ontology that is equally or even more abstract? Since group structural realism is able to dissolve the metaphysical underderdetermination that predates at the object-oriented level, we have sufficient reasons to prefer this metaphysical stance over and above any object-oriented metaphysics. This claim compels us to ask another question: Why should we prefer to solve the underdetermination issue instead of rejecting a more abstract metaphysics? The answer, in my view, is that the underdetermination problem is an epistemic challenge that neglects the mere possibility of endorsing a realist stance with respect to science, whilst the question of whether a metaphysics is abstract or concrete is pragmatic and independent of the question of truth. The point of Section 3 is to argue that no theoretical or metaphysical virtue is available in order to brake the metaphysical underdetermination among Bohmian interpretations at the object-oriented level. If someone argues that a criterion associated with abstractness can be adopted in favor of object-oriented metaphysics and against group structural realism, her argument is objectionable by exactly the same reasons elucidated in Section 3, but extrapolated to the object-structure metaphysical dispute. In fact, any metaphysical virtue, such as abstractness, is not immune to further concerns associated with its normative and regulative nature: How much abstractness one needs? How abstractness is interpreted among different metaphysical views? As is the case for any kind of metaphysical virtue, abstractness is immanent and contextual in the sense that it is highly dependent on the kind of metaphysics adopted, apart from the fact that there is no unique way to specify in any particular situation the precise conditions for this virtue to be satisfied.

Considering these final remarks, together with the rest content of the paper, we conclude that the underdetermination between the three and the high-dimensional worldviews does not pose a serious challenge to interpret BQM in realist terms.

5. Concluding Remarks

After assessing different interpretative Bohmian frameworks in the light of the three/high-dimensional worldviews, we cannot escape from the unfortunate metaphysical underdetermination that infects these alternatives even if we appeal to theoretical or metaphysical virtues. This conclusion does not mean that Bohmian realists have to abandon the project of elucidating the best realist picture in this context. They have at their disposal other two alternatives that block the objection posed by the resulting underdetermination problem: either acceptance of this problem via a pragmatist approach or the appeal to group structural realism that challenges the reality or fundamentality of Bohmian objects and properties in terms of which this problem is based. Considering that we stay within the limits of (non-pragmatist) scientific realism, we endorse the second alternative as opposed to the first one.

APPENDICES

A. The symmetry Group of Bohm's Theory

Some scholars, such as Dürr et al. (1992), take for granted that the symmetry group of BQM is the same to that of classical mechanics: the ten-dimensional connected Lie group known as the *Galilean group* $G_{gal}(3)$ in three-dimensional space and absolute time (see B.1 in the Appendix B). However, the physical scope of this Lie group is constrained by the fact that it is not precisely a dynamical symmetry group but only a spacetime one. This means that $G_{gal}(3)$ only induces spacetime transformations (i.e., boosts, translations and rotations) whose invariances are restricted to mathematical structures defined in (3+1)-dimensional spacetime, such as the guiding equation but not the Schrödinger equation. Although it is true that $G_{gal}(3)$ can be Lie-represented in Hilbert space by means of an appropriate quantisation, many contributions (to be mentioned below) have pointed out that the projective (multi-valued) nature of this representation cannot make the Schrödinger equation a strict invariant of the theory. Not less to say that some quantum properties, such as the spin, are not invariants of spacetime rotations, boosts, and translations. Thus, it seems that the correct symmetry group of BQM should have a richer structure to that of $G_{gal}(3)$.

From the results developed by Greenberger (2001), our first suggestion is to associate the dynamical symmetry group of BQM with that of the non-relativistic limit of standard QM, namely, the *central extension* of $G_{gal}(3)$ (see B.7 in the Appendix B). Contrary to $G_{gal}(3)$, this algebraic extension possesses an additional group element (i.e., a real number corresponding to the generator of mass) that accounts for certain quantum features in the non-relativistic limit, such as mass/charge superpositions. However, it is an eleven-dimensional double-connected Lie group that accepts an appropriate Lie representation in Hilbert space of its group elements that induce (under the Lie representation in the Euclidean three-dimensional space) boosts, spacetime translations and rest-mass differences between inertial observers, with the unfortunate exception of the one that induces spacetime rotations i.e., transformations that arise as a consequence of Lie-representing a subgroup of $G_{gal}(3)$ in the Euclidean three-dimensional space: the special orthogonal group SO(3).

The fact that this group element cannot be 'appropriately' Lie-represented in Hilbert space means that some quantum phenomena, such as 1/2-*spin* particles, cannot de described by the central extension of $G_{gal}(3)$ unless one accepts *double-valued* (or projective) Lie representations. Since the Lie representation of SO(3) in the Hilbert space is double-valued, vectors that are equivalent up to 2π rotations in the three-dimensional Euclidean space are not equivalent in the Hilbert space (they differ up to a minus sign); vectors are, in fact, equivalent in the latter space up to 4π rotations. In physical terms, this means that for every two successive 2π rotations induced by SO(3) along each coordinate axis of the three-dimensional Euclidean space, there are exactly two invariant vectors in the Hilbert space (differing up to a minus sign) associated with two discrete 1/2-spin values.

Fortunately, there is still a better suggestion that may overcome these difficulties. To appropriately define a Lie representation in Hilbert space for the full group (including the 'rotational' element) one must appeal not only to the algebraic extension of $G_{gal}(3)$ but also to its topological extension, namely the *universal cover* of $G_{gal}(3)$ (see B.8 in the Appendix B). Including both the algebraic and topological extensions of $G_{gal}(3)$, the *maximal central extension* of this Lie group is obtained. The need for the topological extension of this group-theoretic structure can be illustrated by appealing to the example of 1/2-spin again. Considering that SO(3) is the (non-normal) subgroup of $G_{gal}(3)$, we can compute the universal cover of SO(3) (in this case, the double cover) and obtain SU(2). The important feature associated with this Lie group, as opposed to SO(3), is that it admits a *single-valued* Lie representation in the Hilbert space, meaning that there exists an unique transformation induced by SU(2) in the Hilbert space associated with the two-fold 1/2-spin values along each coordinate axis. However, although the guiding equation and the Schrödinger equation can be shown to be invariant under the maximal central extension of $G_{gal}(3)$, it is difficult to see how these particular equations, together with the evolution of the Bohmian state, uniquely arise from this group-theoretic structure when it is Lie-represented in the Hilbert space and the configuration space. Under these circumstances, one should be looking for a richer structure to this group maximal extension that can account for both single-valued representations and the complete laws of BQM. This task will be achieved as follows.

Considering that in BQM the quantum state of a N-particle system is not only mathematically defined in the 3N-dimensional configuration space (i.e., the mathematical space of the world particle dynamics), but also in the 3N-Hilbert space (i.e., the infinite mathematical space of wave functions evaluated in the 3N-dimensional configuration space), let us start this task by introducing a condition that the symmetry group of BQM should satisfy: both the Hilbert space and the configuration space must be associated with Lie representation spaces in which the symmetry group of the theory are Lie-represented. Granted this condition, we shall see that the maximal central extension of $G_{gal}(3)$ is a subgroup of a wider symmetry group, i.e., the *inhomogeneous metaplectic group* IMp(M), which is equivalent (up to a maximal central extension) to the dynamical group-theoretic structure of Hamiltonian mechanics, i.e., the *inhomogeneous symplectic group* ISp(M) (see B.4 in the Appendix B). This association shall reveal that, from the group-theoretic point of view, BQM can be interpreted as Hamiltonian theory up to an algebraic-topological mapping. Let us proceed to describe this association in more detail.

As is well known by mathematical physicists, the mathematical formulation of Hamiltonian mechanics follows directly from the invariance and conditions of integrability of Hamilton's equations under coordinate transformations in phase space. From the geometrical point of view, the solutions of Hamilton's equations are integral curves of a vector field associated with the *Hamiltonian flow*, a one-parameter diffeomorphism which takes initial points to final points in phase space retaining the invariance of Hamilton's equations (i.e., canonical transformations). From the group-theoretic point of view (see B.2 and B.3 in the appendix B), however, the Hamiltonian flow is a *symplectic matrix* (up to derivatives) generated by the Lie representation of the *symplectic group* Sp(M) in the cotangent bundle of the M-dimensional Euclidean space (i.e., 2M-phase space), where M=3N for a system of N particles. The essential aspect of the Hamiltonian flow expressed in terms of Sp(M) is that it completely determines the evolution of a classical mechanical system of N particles without the need to appeal to Hamilton's equations. There is, however, one important result that follows from the symplectic structure of Hamiltonian mechanics that we should not leave unnoticed. The Galilean group G_{gal} (3) is a subgroup of a group-theoretic structure associated with Sp(M). Indeed, we can add to Sp(M) space translations and boosts redefined in the 2M-dimensional phase space and obtain the inhomogeneous symplectic group ISp(M). It turns out that $G_{gal}(3)$ can be seen as a subgroup of ISp(M), meaning that the spacetime transformations naturally induced by $G_{gal}(3)$ in the three-dimensional Euclidean space can be seen as transformations defined in the 6N-dimensional phase space.

Alternatively, a similar group-theoretic approach can be adopted in the case of BQM. In the same way that $G_{gal}(3)$ is as a subgroup of ISp(M), the maximal central extension of $G_{gal}(3)$ can be seen as a subgroup of the inhomogeneous metaplectic group IMp(M), where M=3N for a system of N particles. Whilst the inhomogenous part of IMp(M) corresponds to the group of the position-momentum uncertainty relations (i.e., the *polarised Heisenberg group*), the homogenous part of IMp(M) corresponds to the *metaplectic group* Mp(M) (see B.5 and B.6 in the Appendix B). Although a complete understanding of Mp(M) would require more technical details, it simply means that it is a group-theoretic structure that admits a Lie representation in the M-Hilbert space, together with the fact that the one-parameter group of unitary operators and that of symplectic matrices are related in a straightforward manner: Mp(M) may be recovered by computing the double cover of Sp(M). This implies (appealing to a path-lifting property of covering groups) that the classical Hamiltonian flow (i.e., the one-parameter subgroup of Sp(M)), is equal (up to a group homomorphism) to the quantum Hamiltonian flow (i.e., the unique one-parameter subgroup of Mp(M)).

It turns out that the evolution of a Bohmian system (including the evolution of the Bohmian world particle) can be completely determined by the Lie representation of IMp(M) in the M-Hilbert Space and the 2M-phase space (as opposed to the usual formulation in terms of the Schrödinger and guiding equations), where M=3N for a system of N particles. In particular, the Schrödinger equation and the guiding equation arise by representing the inhomogeneous metaplectic group in Hilbert space and phase space, respectively.

B. Definitions of Lie Groups

1. Galilean Lie group

The Galilean group $G_{gal}(3)$ is a 10-dimensional connected Lie group defined by the following semi-direct product:[19]

[19] A group G is a semi-direct product of a normal subgroup S and a subgroup K, denoted by $S \otimes_S K$, if G is expressed as a pair (s, k), where $s \in S$ and $k \in K$, such that the associated product rule is $(s_a,k_i) \cdot (s_b,k_j)= (s_a\, s_b^{k_i},k_i\, k_j)$, where $s_b^{k_i}$ is the action of s_b by k_i.

$$G_{gal}(3) \simeq \mathbb{R}^4 \otimes_S (\mathbb{R}^3 \otimes_S SO(3))$$

The group elements are labeled by g = (b, **r**, **v**, **R**), where b ∈ ℝ, **r** ∈ $\mathbb{R}^3$, **v** ∈ $\mathbb{R}^3$ and **R** ∈ SO(3) are Lie-represented in (3+1)-dimensional spacetime as time translations, space translations, velocity boosts and rotations, respectively.

2. Symplectic matrix

A symplectic matrix s is a 2M × 2M matrix that obeys $s^T \; J s = J$, where J is the block matrix:

$$J = \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix}$$

3. Symplectic Lie group

The symplectic group Sp(M) is a M(2M+1)-dimensional connected Lie group that forms the set of all symplectic matrices with real entries and unit determinant (i.e., a subgroup of the special linear group SL(2M,) under matrix multiplication).

4. Inhomogeneous symplectic Lie group

The inhomogeneous symplectic group ISp(M) is a M(2M+3)-dimensional connected Lie group defined by the semi-direct product of the symplectic group Sp(M) and the translation abelian group A(2M), which is homeomorphic to the Euclidean group $\mathbb{R}^{2M}$:

$$ISp(M) \simeq \mathbb{R}^{2M} \otimes_S Sp(M)$$

The product rule is $(w_a, A_i) \cdot (w_b, A_j) = (w_a \, w_b^{A_i}, A_i A_j)$ where $w_a \, w_b^{A_i} = w_a + A_i \, w_b$ and $w \in \mathbb{R}^{2M}$, A ∈ Sp(M).

5. Polarised Heisenberg Lie group

The polarised Heisenberg group $H_{pol}(M)$ is a simply connected and connected Lie group, usually defined in terms of the multiplicative group of upper triangular (2M+1) × (2M+1) matrices. The underlying manifold is $\mathbb{R}^{2M+1}$ and is topologically defined as the semi-direct product of:

$$\mathrm{H_{pol}(M)}\ \ \mathbb{R}^{2M+1} \otimes_S \mathbb{R}^M$$

whose expression is due to the existence of a non-degenerate alternating $\mathbb{R}$-bilinear form $\mathbf{p} \cdot \mathbf{r}'$: $\mathbb{R}^{2M} \otimes_S \mathbb{R}^{2M} \to \mathbb{R}$ such that the product rule of the group is $(t, \mathbf{z}) \cdot (t', \mathbf{z}') = (t + t' + \mathbf{p} \cdot \mathbf{r}', \mathbf{z} + \mathbf{z}')$ where $\mathbf{z} = (\mathbf{r}, \mathbf{p}) \in \mathbb{R}^{2M}$, $t \in \mathbb{R}$.

6. Metaplectic Lie group

As defined in de Gosson (2001, Ch. 6), Mp(M) is a connected Lie group defined by the set of all products of a finite number of quadratic Fourier transforms of the type:

$$S_{W,m}\Psi(\mathbf{r}) = \left(\frac{1}{2\pi i}\right)^{M/2} i^m \sqrt{\left|Hess_{\mathbf{r},\mathbf{r}'}(-W)\right|} \int_{\mathbb{R}^M_{\mathbf{r}}} e^{iW(\mathbf{r},\mathbf{r}')}\, \Psi(\mathbf{r}')\, d^n\, \mathbf{r}'$$

where the integers m are defined over Mp(M) by $m(S_{W,m})=\arg(Hess_{\mathbf{r}\cdot\mathbf{r}'})(-W))$ mod 4. They are the *Maslov index* of the quadratic form W (i.e., a generating function in phase space).

7. Central Extension

An extension of a group H by a group N is a group G with a normal subgroup M such that M ≃ N and G/M ≃ H. Thus, there is a short exact sequence of groups:

$$1 \to \mathrm{N} \overset{\alpha}{\to} G \overset{\beta}{\to} H \to 1$$

where $\alpha : N \to G$ is injective, $\beta : G \to H$ is surjective, and the image of α is the kernel of β. An extension is called a central extension if the normal subgroup N of G lies in the center of G (the set of elements that commute with every element of G).

8. Symplectic covering groups

The symplectic group Sp(M) generates a set of connected Lie groups $\mathrm{Sp}_2(\mathrm{M})$, $\mathrm{Sp}_3(\mathrm{M})$, ... ,$\mathrm{Sp}_q(\mathrm{M})$, ... , $\mathrm{Sp}_\infty(\mathrm{M})$, called q-order covering groups, such that for every q there is a group homomorphism (covering homomorphism) $\pi_{\mathbf{q}} : \mathrm{Sp}_q(\mathrm{M}) \to \mathrm{Sp}(\mathrm{M})$ with the following properties:

If $q < \infty$

1. $\pi_{\mathbf{q}}$ is surjective and q-to-one. In other words, π_q^{-1} ($\mathbb{I}$) contains q elements
2. $\pi_{\mathbf{q}}$ is continuous and a local diffeomorphism: $\forall s \in$ Sp(M), $\exists U_S$ an open neighborhood, such that π_q^{-1} $(U_S) = \cup_k U_k$, where U_k is the neighborhood of $S_k \in \mathrm{Sp}_q$(M) and $U_i \cap U_j$, $\forall$ i, j.

If $q = \infty$

1. Sp_∞(M) is called the universal cover and π_∞^{-1} ($\mathbb{I}$) = ($\mathbb{Z}$,+) the fundamental group of Sp(M). Since π_∞ is a homomorphism and sends ($\mathbb{Z}$,+) to the identity element, Sp_∞(M) is simply connected.

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