HUME[1] AND DESCARTES[2] ON SKEPTICISM WITH REGARD TO DEMONSTRATIVE REASONING

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Resumen

Los comentarios sobre la sección "Of scepticism with regard to reason" en A Treatise on Human Nature de David Hume (Libro I, Parte IV, sección 1) se han concentrado en el argumento según el cual, la falta inicial de convicción certera acerca de la conclusión de una inferencia cualquiera disminuye gradualmente a cero. Sostengo que Hume ofrece este famoso argumento solamente después y como corolario de un argumento escéptico mucho más interesante acerca del razonamiento deductivo, el cual aparece al principio de esta sección. Me concentro en este primer argumento, señalo sus raíces cartesianas, y establezco una distinción entre dudas comunes y una duda escéptica radical acerca de la interferencia inevitable, en nuestros razonamientos deductivos, de facultades mentales falibles. Hume sugiere que, en la vida cotidiana y en la ciencia, las soluciones a las dudas acerca de la falibilidad humana, en sí mismas se apoyan en razonamientos de carácter causal: si hemos aplicado correctamente una regla inferencial o no, en una instancia específica, es en sí misma una conclusión acerca de lo que Hume llama "matters of fact". Por lo tanto, está basada en el único tipo de evidencia que tal conclusión puede poseer. El argumento brillante de Hume invierte negativamente la confianza creciente que normalmente adquirimos sobre la base de tal razonamiento causal. Una vez que tomamos en cuenta, desde el punto de vista escéptico radical, que en nuestros intentos de mejorar y evaluar razonamientos deductivos usamos meramente razonamientos causales, se cierra la posibilidad de ajustar progresivamente el ejercicio de nuestras facultades mentales a una supuesta validez objetiva de las reglas de deducción. El aumento de la confianza en nuestras inferencias está basado en métodos causales, por lo tanto resulta en "la adición de nuevas probabilidades", y no existe una transición gradual desde la pro-

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babilidad más alta hasta el conocimiento certero demostrativo. De esta manera, Hume llega a una duda escéptica radical acerca de la posibilidad de lograr conocimiento demostrativo genuinamente certero. A diferencia de Descartes, Hume obtiene esta duda radical sin apelar a un elemento externo a nuestras facultades mentales, tal como el todopoderoso que engaña en la Primera Meditación de Descartes.

PALABRAS CLAVE: Hume - Descartes - escepticismo - razonamiento demostrativo - razonamiento causal - conocimiento a priori - reglas de inferencia - probabilidad

Abstract

Commentaries on Hume's Treatise 1.4.1, "Of scepticism with regard to reason," have focused on the argument that an initial lack of certainty concerning the conclusion of an inference gradually diminishes to zero. In my view, Hume offers this famous argument only after, and as corollary to, a far more interesting skeptical argument concerning demonstrative reasoning, which occurs at the very beginning of Treatise 1.4.1. I focus on this neglected argument, point to its Cartesian roots, and draw a distinction between ordinary doubts and a radical skeptical doubt about the inevitable interference of fallible faculties in our demonstrative inferences. Hume suggests that, in common life and science, solutions to ordinary doubts concerning human fallibility themselves rely on causal reasoning—that we have applied inferential rules correctly in any given instance is a "matters of fact" conclusion, thus it is supported by the only kind of evidence that such a conclusion can have. Hume's argument brilliantly reverses the force of the increased confidence normally acquired on the basis of such causal reasoning. Once we have realized, in the radically skeptical frame of mind, that in our attempts to improve and evaluate demonstrative reasoning we use merely causal reasoning, there is no longer the hope of a progressive adjustment of the exercise of our faculties to an assumed objective validity of our demonstrative rules.
 The increase in assurance by causal methods only amounts to "the addition of new probabilities," and there is no gradual transition from probability to demonstratively certain knowledge. Hume thereby reaches a radical skeptical doubt regarding the possibility of our ever attaining genuinely certain demonstrative knowledge — and, unlike Descartes, he reaches this doubt without the external device of an all-powerful deceiver.

KEY WORDS: Hume - Descartes - Skepticism - Demonstrative reasoning - Causal reasoning - A priori knowledge - Probability - Rules of inference

At *Treatise* Book One, Part Three, Section One and Two, Hume distinguishes between two kinds of philosophical relations and calls them, respectively, "Knowledge" and "Probability." This distinction follows Locke's terminology and Locke's model of a phenomenological apprehension of image-like items before the mind.[3] Nonetheless, Hume provides an orig-

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[3] John Locke, *An Essay concerning Human Understanding*, Peter H. Nidditch edition (Oxford University Press, Clarendon Edition, 1975) (abbreviated *Essay*), Book IV.

inal characterization of each of these two types of cognition. The dichotomy goes back at least to Descartes, who draws it against the background of an intended intellectualist conception of the apprehension of ideas. In Rule Two of the *Rules for the Direction of the Mind* (*Regulae*), Descartes claims that all "Scientia" (systematic knowledge based on indubitable foundations) is evident and certain. Descartes here contrasts Scientia with probable cognition, and regards arithmetic and geometry as the paradigm of the former.[4] In the *First Meditation*, after entertaining the dream and the powerful deceiving God hypotheses, Descartes adopts the stringent policy of regarding all his habitual opinions as if they were false. Descartes justifies this policy by pointing out that although most of his previous beliefs are in need of a solid foundation, they are highly probable.[5] This is one central reason why they are so powerful, so that, in particular, "hyperbolic" skeptical hypotheses are needed to counteract them: "I shall never get out of the habit of confidently assenting to these opinions, so long as I suppose them to be what in fact they are, namely highly probable opinions — opinions which, despite the fact that they are in a sense doubtful, as has just been shown, it is still much more reasonable to believe than to deny. In view of this, I think it will be a good plan to turn my will in completely the opposite direction and deceive myself, by pretending for a time that these former opinions are utterly false and imaginary."[6]

Hume develops his own distinction between knowledge and probability as two different ways in which we reflectively compare ideas,

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[4] Rule Two prescribes: "*We should attend only to those objects of which our minds seem capable of having certain and indubitable cognition.*" At the beginning of the explanation of this rule, Descartes writes: "All knowledge is certain and evident cognition. Someone who has doubts about many things is no wiser than one who has never given them a thought; indeed, he appears less wise if he has formed a false opinion about any of them. Hence it is better never to study at all than occupy ourselves with objects which are so difficult that we are unable to distinguish what is true from what is false, and are forced to take the doubtful as certain; for in such matters the risk of diminishing our knowledge is greater than our hope of increasing it. So, in accordance with this Rule, we reject all such merely probable cognition and resolve to believe only what is perfectly known and incapable of being doubted." At the end of the next paragraph, Descartes adds: "Accordingly, if my reckoning is correct, out of all the sciences so far devised, we are restricted to just arithmetic and geometry if we stick to this Rule." Descartes, *Rules for the Direction of the Mind*, (composed around 1628), Cottingham et al, Vol. I, pp. 10-11.

[5] In my book manuscript, *Ideas, Evidence, and Method: Skepticism and Naturalism in Hume's Epistemology*, I discuss the differences between Descartes's and Hume's notions of probability. The differences become very clear, in particular, when we appreciate Newton's influence on Hume.

[6] Descartes, *Meditations on First Philosophy*, Cottingham et al, Vol. II, p. 15.

impressions or objects, and thereby establish that two kinds of philosophical relations obtain. By means of the first method of comparing items we
can attain the "full certainty" (T 1.3.1.6 / SBN p. 72) of knowledge, precisely because this method relies solely on the direct inspection of features
intrinsically contained in the items related.[7] Thus, at T 1.3.3.2 / SBN p.
79, in the context of arguing that the maxim "*whatever begins to exist,
must have a cause of existence*" is not certain, Hume writes: "All certainty arises from the comparison of ideas, and from the discovery of such relations as are unalterable, so long as the ideas continue the same." In
particular, if the intrinsic content of items are phenomenologically presented as resembling or contrary to each other, or as possessing different degrees of a quality, or as bearing a certain proportion of quantity or
number, then these contents have these relations so long as they are
apprehended as stable. The intrinsic content of items might be presented in different events of apprehension, for example, in the sequence of
steps of a mathematical proof. Yet, if the contents in the sequence are
apprehended as qualitatively identical or as quantitatively identical (that
is, as equal in magnitude, not as numerically identical), as one contained
within the other, or as not contained or excluding one another, such relations among the contents are apprehended with certainty. Moreover, I can
determine solely on the basis of an intrinsic content now ostensively present, prior to experiencing any other particular content, which intrinsic
features a new experience would have to have in order, for example, to
resemble the one now before the mind. Phenomenologically presented
intrinsic features of the relata make the first kind of philosophical relations certain, necessary, and a priori.[8]

By contrast, in order to determine whether the second kind of philosophical relations obtain, we are required to go beyond the intrinsic features of contents that are or have been apprehended. There are three
relations of the second kind. One is the numerical identity (including temporal continuity) of contents that are in fact apprehended in a sequence
of separate events of apprehension. The other two are situations in time
and place, and causation. Hume discusses these relations at *Treatise* 1.3.2,
under the title "*Of probability; and of the idea of cause and effect*." "Mat-

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[7] In my paper "Causation as a Philosophical Relation in Hume," *Philosophy and
Phenomenological Research*, Vol. LXIV, No. 3, May 2002, pp. 499-545, I make this thesis the central premise of my interpretation of Hume's distinction between two kind
of philosophical relations.

[8] In *Enquiry*, 4.6 / SBN p. 27, Hume first uses the word "a priori" negatively,
to claim that "matters of fact and existence" are not a priori.

ters of fact and existence" in the *Enquiry* correspond to those relations which, according to the *Treatise* (T 1.3.1.1 / SBN p. 69), "may be chang'd… without any change on the objects themselves or on their ideas." This contrasts with the first kind of relations, which, as we have just seen, remain unalterable as long as the ideas themselves are unchanged. As an example of the first kind, Hume gives the relation of equality between the angles of a triangle and two right angles, which, Hume says, is derived from the idea of a triangle, and "is invariable, as long as our idea [of a triangle] remains the same" (T 1.3.1.1 / SBN p. 69). By contrast, "the relations of *contiguity* and *distance* betwixt two objects may be chang'd merely by an alteration of their place, without any change on the objects themselves or on the ideas" (*Treatise,* ibid.). Hume adds (T 1.3.2.2 / SBN p. 74): "There is nothing in any objects to perswade us, that they are either always *remote* or always *contiguous*." This is a very important point, since it reveals that by "nothing in any objects" Hume means no phenomenologically presented intrinsic feature or content of the items before the mind. Thus, for example, two adjacent reddish spots in the visual field can be moved to any non-adjacent positions without changing their intrinsic features (colors), but their relation of resemblance (in color) cannot be changed without changing the ideas themselves. Similarly, the relationship between the angles of a triangle and two right angles can also not be changed without changing the idea of a triangle itself (here its intrinsic geometrical features), but changing its position in space (relative to other geometrical figures, say) leaves its own intrinsic features completely unchanged. Since we cannot determine whether the second kind of philosophical relations obtain solely by relying on the qualitative identity or containment of intrinsic features of the relata, these relations are extrinsic, contingent, and a posteriori. They depend on features of the relata that make the relata separable from, that is, independent of each other.

Hume marks a further distinction within each of the two kinds of philosophical relations. Within the first kind there is either intuitive or demonstrative knowledge. We can assess that resemblance, contrariety, and degrees in quality obtain by means of a single act of comparison. Hume writes (T 1.3.1.2 / SBN p. 70): "Three of these relations are discoverable at first sight, and fall more properly under the province of intuition than demonstration." Hume continues in the same paragraph by pointing out that qualitative resemblance among contents "at first strike[s] the eye, or rather the mind; and seldom requires a second examination." The comparison of ideas in simple cases of the fourth of the a priori relations — proportions of quantity or number — can likewise amount to intuitive knowledge (T 1.3.1.3 / SBN p. 70): "We might proceed,

after the same manner, in fixing the *proportions* of *quantity* or *number*, and might at one view observe a superiority or inferiority betwixt any numbers, or figures; especially where the difference is very great and remarkable." The phrases Hume uses in characterizing intuitive knowledge in these passages ("at first strike the [mind]," "seldom requires a second examination," "at one view") reveal that he has in mind a certainty which is achieved by a discrete, temporally bound act of apprehension of evidence. He also uses phrases such as "single consideration" and "comprehended in an instant" (T 1.3.1.3 / SBN p. 70) to the same effect. Most of mathematics, however, involves complex cases of relations of proportion, which require the use of inferential reasoning. Nevertheless, mathematical inferences are based solely on the intrinsic content of ideas (in particular, on relations of proportion among geometrical figures or among discrete units), and can thus amount to demonstrative necessary knowledge and provide full certainty.

In the discussion of probable relations, Hume again refers to single acts of apprehension as opposed to inferential reasoning. The contrast is between, on the one hand, what Hume here calls "perception," which consists in the apprehension by a single act of the mind of immediately given impressions and their immediately given extrinsic relations —such as the relations of time and place— and, on the other hand, reasoning based on sensory impressions and memories of them. Hume writes (T 1.3.2.2 / SBN p. 73): "When both the objects [which we compare] are present to the senses along with the relation, we call *this* perception rather than reasoning." The causal inference to the unobserved is the most central of probable inferences and always provides less than full certainty.

Hume's distinction between intuitive and demonstrative (a priori) knowledge, like the distinction between knowledge and probability, has an illustrious history. The most immediate antecedent is Locke's distinction in *An Essay concerning Human Understanding*, Book IV, where Locke uses the same terminology of "intuition" and "demonstration" (although Locke's examples of intuitive knowledge are different from Hume's).[9] Locke's division has in turn immediate Cartesian roots. In the *Regulae*, Descartes gives a prominent place to the contrast between "mental intuition" (immediate mental apprehension) and deduction, starting with Rule Three.[10] A more direct influence on Locke might be Descartes's *Fifth Meditation* and the *Second and Fourth Replies* to the charge of circularity in the proof of the trustworthiness of clear and dis-

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[9] Locke, *Essay*, Book IV, Chapter II.
[10] Descartes, *Rules for the Direction of the Mind*, Cottingham et al, Vol I, pp. 13f.

tinct ideas. In these later writings Descartes also suggests, as in the *Regulae*, that at least some cases of apprehension of clear and distinct ideas by a single act of an attentive mind (the *Regulae*'s mental intuition) are certain, whereas the apprehension of long demonstrations is not. (I return to this Cartesian thesis below.) However, Locke gives what I call a "sensible" version of Descartes's intellectualist conception of a priori knowledge, since Locke recasts it in terms of the apprehension of particulars in the style of sensory perception and the imagination. Hume adopts and radicalizes this Lockean revision.[11]

The central target of Hume's *Treatise* 1.4.1, entitled "Of scepticism with regard to reason," is, in my view, any "chain of reasoning,"[12] which goes beyond a simple apprehension by a single act of the mind. Hume is not primarily concerned with either intuitive knowledge or with what in *Treatise* 1.3.2 he calls "perception." The first sentence of *Treatise* 1.4.1 reveals that Hume's key concern is inferential reasoning (T 1.4.1.1 / SBN p. 180): "In all demonstrative sciences the rules are certain and infallible." In addition, Hume illustrates his initial points with the case of "discoveries" made by an "Algebraist" or a "Mathematician" by means of *proofs*, and also refers to "long numerations." These illustrations also disclose that Hume follows Descartes in taking mathematics as the paradigm of a priori rational knowledge.[13] This section offers first a radical skeptical argument concerning demonstrative knowledge and, on its basis, proceeds with a second radical skeptical argument concerning probable inference, thereby casting radical doubts regarding any idea whatsoever which is based on the employment of inferential reasoning. Thus, the word "reason" in the title refers both to demonstrative and probable inferential reasoning.

Both skeptical arguments, concerning demonstrative and probable reasoning respectively, proceed by taking into account the fallibility of our faculties. The appeal to the limitations and deceptive nature of human faculties in order to generate radical skeptical doubts also has Cartesian roots. Ordinary doubts, in contradistinction to radically skeptical doubts, always leave room for correction and improvement. For, in common life and science, we assume that we can continually approximate

[11] I argue for these theses in my book manuscript mentioned in note 5 above.

[12] For Hume's use of this expression in talking about algebra and arithmetic, see T 1.3.1.5 / SBN p. 71.

[13] Descartes takes mathematics to be such a paradigm to the detriment of logic. I discuss the extent of Descartes's neglect of logic and the consequences for Descartes's conception of a priori knowledge in my work cited in note 5 above.

the exactitude and certainty of ideal rational inferences. In these same contexts, we take such inferences objectively to confer certainty on their conclusions (in the case of deductive inference), or to support them to a certain degree (in the case of probable inference). Radical skeptical doubts, by contrast, do not leave room for the idea of a positive progress towards accurately apprehending objective relationships of dependence between premises and conclusions. In *Treatise* 1.4.1 Hume, like Descartes, raises radical skeptical rather than ordinary doubts about the employment of our faculties. Thus, despite Hume's rejection of Descartes's ideal of a pure intellect and of Descartes's conception of belief as an act of the will,[14] Descartes's skeptical doubts about our faculties are a likely source of Hume's approach. It might appear, therefore, that Hume is not proposing an original form of radical skepticism at *Treatise* 1.4.1. However, as we shall see, Hume here uses the Cartesian legacy in a quite original way. Moreover, we can find lasting echoes of Hume's skeptical argument concerning a priori reasoning even in twentieth century philosophy.[15] Let us assess Hume's contribution by considering first Descartes's own radically skeptical argument regarding a priori demonstration.

In the *First Meditation*, Descartes raises the possibility that even the most trustworthy of our faculties might deceive us all the time. After the dream argument, which concerns our knowledge by means of the senses of the existence and character of corporeal nature, but before the Deceiving God hypothesis, Descartes writes: "So a reasonable conclusion from this might be that physics, astronomy, medicine, and all other disciplines which depend on the study of composite things, are doubtful; while arithmetic, geometry and other subjects of this kind, which deal only with the simplest and most general things, regardless of whether they really exist in nature or not, contain something certain and indubitable. For whether I am awake or sleep, two and three added together are five, and a square has no more than four sides. It seems impossible that such transparent truths should incur any suspicion of being false."[16] However, in the following paragraph, Descartes introduces the hypothesis of a deceiv-

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[14] These are two tenets of Cartesianism the rejection of which might also be guiding Hume's discussion in *Treatise* 1.4.1. For example, Hume emerges from the radical skeptical exercise of this section by reasserting his theory of belief, which is opposed not only to Descartes's view of belief as an act of the will but also to views that make belief dependent on reasoning that is not mixed with impressions and natural propensities.

[15] I believe there are fruitful connections to be found between Hume's argument here and Wittgenstein's discussion of following a rule. However, the consideration of such connections must be left for another occasion.

[16] Descartes, *Meditations on First Philosophy*, Cottingham et al, Vol. II, p. 14.

ing God, which he later recasts as the hypothesis of a malicious demon. This hypothesis raises the possibility that we might be constantly deceived in the exercise of our faculties, even in acts of apprehension of the simplest and most transparent ideas of mathematics. Thus, Descartes writes: "What is more, since I sometimes believe that others go astray in cases where they think they have the most perfect knowledge, may I not similarly go wrong every time I add two and three or count the sides of a square, or in some even simpler matter, if that is imaginable?"[17] At this point, therefore, in terms of Descartes's distinction in Rule Two of the *Regulae* between Scientia and probable cognition, *all* cognition —even the simplest such as adding two and three— amounts only to highly probable opinion. For, there is a possibility of systematic deception in the exercise of even our most reliable faculty of the pure intellect. Neither claims about the external world nor mathematical claims have been shown to constitute Scientia, since no such claims have been shown to be indubitable. Hume also first reduces certainty, as we shall see, to probable opinion, but without using a device (like Descartes's powerful deceiver) external to our own faculties.

In the *Fifth Meditation*, and in addressing the circularity objection in the *Second* and *Fourth Replies*,[18] Descartes appears to correct the *First Meditation*'s suggestion that the skeptical doubt about purely intellectual ideas includes clear and distinct ideas apprehended in a single act of the mind. In these subsequent texts, Descartes directs the reader's attention away from such a single act of apprehension and claims instead that the proof of the existence of a benevolent God is meant to secure the certainty of long demonstrative inferences or of demonstrative inferences to which we are not now attending. In the *Regulae*, Descartes had already raised the issue of the need to guarantee the certainty of long deductions. However, in that earlier work Descartes offers only ordinary doubts about demonstrative reasoning. There Descartes attributes the source of error in long deductions to memory, but he finds a remedy in a kind of training that, according to the *Regulae*, can improve our apprehension of long proofs. Rule Eleven, for example, shows that deduction can become certain when we can reduce its apprehension to a single intuitive act of the mind: "If, after intuiting a number of simple propositions, we deduce something else from them, it is useful to run through them in a continuous and

[17] Descartes, *Ibid*.
[18] Descartes, *Author's Replies to the Second Set of Objections*, "*Thirdly*" and "*Fourthly*", Cottingham et al, Vol. II, pp. 100f; *Author's Replies to the Fourth Set of Objections*, *op. cit*., p. 171.

completely uninterrupted train of thought, to reflect on their relations to one another, and to form a distinct and, as far as possible, simultaneous conception of several of them."[19] In the explanation of this Rule, Descartes says that the rule is needed in order to overcome the inherent fallibility of memory. Yet, to assume that running through a proof in a continuous and uninterrupted train of thought will solve the problem of the fallibility of memory is to grant that we cannot be deceived by our faculties all the time —that we can improve their performance by applying tests, undergoing training, and so on— contrary to the radical skeptical doubt of the Meditations.[20] The *Fifth Meditation* and the *Replies*, unlike the *Regulae*, provide a guarantee for the certainty of long demonstrations that is external to the exercise of the faculties themselves. The doubt inevitably brought about by going beyond self-evident single acts of the mind (clear and distinct ideas) can only be eliminated when we attend to the proof that God exists and is not a deceiver. By appealing to this external device, Descartes reveals that he is still concerned, as in the *First Meditation*, with the possibility of being deceived all the time in the exercise of our faculties (with philosophical hyperbolic doubt), even though now the doubt seems to be restricted to demonstrative inference.

In addition to posing radical skeptical doubts about demonstrative reasoning, Descartes can be taken to be a source for Hume's *Treatise* 1.4.1 in another respect. Descartes invites the consideration of psychological, subjective, and empirical (causal) aspects of the human knower, which undermine the certainty of the pure intellect, and thus of a priori demonstrative knowledge. For Descartes calls attention to the fact that we are required to employ a faculty whose exercise, whether part of the intellect or not, can be fallible (even for the most attentive and well trained mind) precisely because we must go beyond a present mental intuition. The need to rely on the fallible performances of memory arises from the fact that an inference takes place in successive separate moments of time, through separate acts of apprehension. In calling attention to the time-

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[19] Descartes, *Rules for the Direction of the Mind*, Cottingham et al, Vol. I, p. 37.

[20] An interpretative puzzle concerning Descartes's conception of memory arises both in the *Meditations* and the *Regulae*. For, it is not obvious how Descartes could allow that a faculty of memory which is related to the body through the senses and the corporeal imagination can play a significant role in mathematical proofs. Thus, memory as used in demonstrations might be plausibly taken to be part or share attributes of the pure intellect. If a memory could in principle retrieve (although perhaps not always in a reliable way) the original immersion in an intellectual certain apprehension of clear and distinct ideas, such a memory must somehow be able to apprehend in the same way the same items the intellect apprehends.

bound aspect of inferences, Descartes moves away from focusing on the objectivity and non-empirical character of demonstrative proofs and the rules governing them — contrary to what Leibniz, for example, will later very forcefully emphasize. In spite of this invitation to psychologism regarding demonstrative inference, however, Descartes also argues in the opposite direction. In the *Fifth Meditation*, for example, Descartes claims that we can know that there are immutable and eternal mathematical natures, which are independent of the human mind, precisely because we can demonstrate that certain properties necessarily belong to mathematical essences, whether we wish it or not. By insinuating that demonstrations governed by rules allow us to have access to some kind of platonic essences or forms, Descartes suggests that he acknowledges the objectivity and non-psychological character of necessary a priori demonstrations.

Let us now consider the details of Hume's first skeptical argument. In talking about fallible "faculties" in the plural throughout *Treatise* 1.4.1, Hume remains neutral whether his argument about demonstrative reasoning concerns the pure intellect versus other faculties such as memory or the imagination. Hume confronts the reader with paradoxes about reasoning conceived either as a Cartesian pure intellect which apprehends eternal an immutable truths independent of the mind, or as a Lockean and Humean series of phenomenological acts of apprehension of particular items in the style of sensory perception or the imagination. Hume attempts to produce a challenge general enough to call into question the certainty of inference as such, in whatever way we conceive it.

Hume declares that "in all demonstrative sciences the rules are certain and infallible." This claim does not mean that he endorses a rationalist conception of the objectivity of mathematical rules, according to which rules represent objectively valid relationships of dependence between premises and conclusions.[21] Rather, Hume means that in common life and science— in particular, in mathematical calculations —we assume that the method of establishing claims by demonstrative reasoning provides certain knowledge. Hume initially refers to the fact that even in common life and science we can raise ordinary doubts about the certainty of our con-

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[21] In my paper "Causation as a Philosophical Relation in Hume," quoted in note 7 above, I argue that the principle of non-contradiction, to which Hume appeals in the *Enquiry* in order to characterize relations of ideas, reduces to a purely sensible phenomenological mode of apprehension. In other words, in Hume such a principle does not have a validity beyond the relations of ideas we can empirically and contingently establish according to our perceptual and imaginative powers. Hume's epistemology lacks the resources to accord to the principle of non-contradiction the objective validity attributed to it by, for example, Leibniz and Kant.

clusions, given that we apply the rules only by means of "fallible and uncertain faculties." The rules are assumed to be certain and infallible, "but when we apply them, our fallible and uncertain faculties are very apt to depart from them, and fall into error" (T 1.4.1.1 / SBN p. 180). No matter how experienced mathematicians (or merchants, or accountants) might be, they realize that they cannot be truly certain in their own calculations, because they are aware that they work with fallible faculties.

In common life and science ordinary reasoners and experts naturally attempt to assess and improve the reliability of the application of inferential rules. Hume observes that the standards for evaluating the application of inferential rules by fallible faculties are in fact those of causal reasoning. In common life and science we take into consideration past experience in estimating the degree of reliability of the inferential abilities of the reasoner. We also subject inferences to several tests, which include repeatedly running over the inferences and seeking intersubjective agreement. The natural and gradual increase in confidence after obtaining positive results (T 1.4.1.2 / SBN pp. 180-81): "…is deriv'd from the constant union of causes and effects, according to past experience and observation." Here, the constant union of causes and effects is the observation of a conjunction of performances of reasoners, on the one hand, and positive test results of the above kind associated with those performances, on the other. If we find a constant or near constant conjunction of past experiences of drawing conclusions and of having them certified by careful examination, then we can in fact increase our confidence in our inferential abilities, and thus in our present or next inferences. The performance of a priori demonstrations, as of any other inference, is thus subject to the control of causal reasoning. This control takes place because we need to evaluate the *application* of inferential rules by means of fallible human faculties. The event of application of rules is itself a causal process, as it consists in the application of what is taken to be a general rule in order to generate, at a particular time, a particular conclusion from particular premises. This is in part what Hume means when he writes (T 1.4.1.1 / SBN p. 180): "Our reason must be consider'd as a kind of cause, of which truth is the natural effect; but such-a-one as by the irruption of other causes, and by the inconstancy of our mental powers, may frequently be prevented."

In this way, Hume initially records the ordinary doubts and solutions to these doubts that take expression in common life and science when an "Algebraist", a mathematician, a merchant, or an "accomptant" evaluates his or someone else's demonstrative inferences. This initial discussion can be regarded as a causal counterpart of Descartes's ordinary

doubts concerning human performances of generating deductions and Descartes's non-skeptical prescription of exercising one's ability to reduce a deduction to a mental intuition in the *Regulae*. And, like Descartes in the *Meditations*, Hume then turns these ordinary doubts into a radical skeptical doubt concerning demonstrative knowledge. This occurs when, contrary to the increase in confidence naturally acquired in common life and science, Hume points out that this increase is brought about by a causal reasoning concerning our fallible faculties, therefore, it amounts only to the addition of new probabilities, never to certainty (T 1.4.1.2 / SBN pp. 180-81): "Now 'tis evident, that this gradual encrease of assurance is nothing but the addition of new probabilities, and is deriv'd from the constant union of causes and effects, according to past experience and observation." In this way, Hume implicitly raises the possibility, as Descartes does explicitly, that our faculty of reasoning might deceive us at any time. More precisely, Hume dooms to failure the attempts to correct and improve the use of the faculty of reasoning (by causal reasoning) in order to gain the certainty expected of mathematical demonstrations. Hume thereby reaches a radical skeptical doubt regarding the possibility of our ever attaining genuine certain demonstrative knowledge — and he reaches this doubt without the external device of an all-powerful deceiver.

By contrast, mathematicians or accountants, for example, when engaged in evaluating their own results —even if they take into consideration the fallibility of their faculties— would not reach Hume's skeptical conclusion. In common life and science, increased assurances gained by successful past performances, by careful checking of our proofs, by intersubjective agreement, and so on, point to the possibility of gradually approximating the certainty of demonstrative proofs. Hume brilliantly reverses the force of such increased assurances: once we have realized, in the radically skeptical frame of mind, that in our attempts to improve demonstrative reasoning we use merely causal reasoning, there is not any longer the hope of a progressive adjustment of the exercise of our faculties to an assumed objective validity of the rules governing the inference. The increase in assurance by causal methods only amounts to "the addition of new probabilities." There is no gradual transition from probability to certain knowledge (T 1.4.1.3 / SBN p. 181): "But knowledge and probability are of such contrary and disagreeing natures, that they cannot well run insensibly into each other; and that because they will not divide, but must be either entirely present, or entirely absent."

Hume is right that knowledge and probability "cannot well run insensibly into each other", because probable inference, according to

Hume, no matter how successful, introduces uncertainty: it can never establish the relations of qualitative identity or difference, or equality or precise difference of magnitude, or containment or exclusion of intrinsic features of ideas demanded by demonstrative knowledge. The state of our conviction in knowledge, by definition, must be entire or full certainty, whereas our conviction regarding conclusions of probable reasoning is always less that entire. Certainty is exclusively entire or full conviction; otherwise it is not certainty and not knowledge. Hume's radical skeptical conclusion to the effect that what we initially took to be demonstrative knowledge degenerates into probable inference is deeply paradoxical. The paradox arises because, as previously characterized by Hume himself, and by Locke, Descartes, Leibniz, and others, "knowledge and probability are of such contrary and disagreeing natures," knowledge cannot degenerate into probability — yet Hume has shown that it does. With this paradox Hume intends to show, I believe, that despite the fact that we commonly regard the rules of mathematics as certain and infallible, such presumed certainty and infallibility signify nothing to us since we are prisoners of our own faculties. The exercise of fallible faculties constitutes the only actual realization of the rules: our inferences must always be tainted by the tools we use in applying the rules of inference, whatever the objective status of the rules. And our assessment of this application must always be based on experience.

Thus, Hume brings to the fore an unintended consequence of Descartes's consideration of inference as a series of successive events of apprehension, each of which takes place at a separate moment of time and is in this sense independent from the others. The application of allegedly objectively valid rules to premises and conclusions that are separately apprehended introduces causal factors that might disturb the correctness of the inference. For the series of events of applying the rules is itself an empirical causal process: we need to generate, at a particular time, a particular conclusion from particular premises on the basis of an understanding by our faculties of a general rule, and this understanding, in turn, is based on previous applications of the rules. No wonder, then, that in common life and science all attempts at improving our inferences rely on empirical probable reasoning concerning intersubjective agreement, past performances, and so on.

According to the Cartesian-Lockean characterization of a demonstration as a series of intuitive steps, we would expect that in an arithmetical demonstration (in contradistinction to a probable inference) we would be able to asses whether our inference is correct or not by successively comparing the ideas in the different steps through a series of indi-

vidual intuitive acts of apprehension. In this way, according to Hume's characterization of the first kind of philosophical relations, we would expect to be able phenomenologically to discover which ideas are quantitatively contained in one another, are different to a certain quantitative degree, are identical with or exclude one another. However, we radically change our perspective when we realize that the acts of apprehension of premises and conclusion are temporally separated events, so that, even if intuitive acts of apprehension are infallible, demonstrative inference is not. Now we view the premises and conclusion as themselves independent from one another, for the steps in question are generated in a temporal sequence of separate events of apprehension.

By contrast, intuitive knowledge seems so far to be wholly immune from radical skeptical doubt. So it might seem, for example, that we could fail in our application of a rule only in those cases in which we do not recognize that the premises and conclusion constitute an instance of that rule. And this failure would occur only in those cases in which we could not grasp the inference and the rule all at once in a single intuitive act of apprehension. For some very simple inferences governed, for example, by Modus Ponens, we might intuitively apprehend both the inference and the rule simultaneously, so as to ascertain that the form of one is contained in or identical with the form of the other. The conception of rules as reducible to conditional propositions would then appear to satisfy the desideratum of achieving an intuitive apprehension of rules of inference.

However, Hume does not consider that we might intuitively apprehend a rule or any other simple inference as a conditional, and he certainly does not conceive the application of a rule to a given inference as a matter of taking notice of a shared form. At *Treatise* 1.4.1 Hume is concerned mainly with arithmetic and algebra. In arithmetic the only kind of rule Hume mentions is a phenomenological apprehension of an ostensively given operation of making units coincide or overlap with one another by an intuitively presented one-to-one correspondence (T 1.3.1.5 / SBN p. 71) — there is no question at all of any formal notion of one-to-one correspondence as in Frege. Moreover, when Hume starts by claiming that in all demonstrative sciences the rules are (taken to be) certain and infallible, he means that, in common life and science, we (take ourselves to) have knowledge of how to make the required transitions from premises to conclusion. In taking the achievements of the demonstrative sciences to depend on assumed infallible rules, we make the same assumption: we know how to proceed, under normal circumstances and given appropriate expertise, from a given step to one that is not yet present before the mind. Nevertheless, in raising the radical skeptical doubt and thus turn-

ing our assurance about demonstrative inference into mere causal conviction, Hume reveals that he is now challenging this very assumption. From this radical skeptical perspective, it is only *after* we have *completed* the inferential sequence involved in the application of even a very simple rule of inference that we could then proceed to reduce this inference to an intuitive act of comparing conditionals. But, from this same perspective, there is neither intuitive nor demonstrative knowledge of how to proceed to a next step that is not yet present before the mind — just as there is no such knowledge in the case of the causal inference to the unobserved. Thus, it is only after we have figured out how to proceed in this way that we can then have intuitive knowledge of even the very simplest case.[22]

In sum, according to the first radical skeptical argument of *Treatise* 1.4.1, when we acknowledge that we apply inferential rules using irremediably fallible faculties, we must take the conclusions of our inferences to be at best probable opinion. Even the best case scenario of improving the exercise of our faculties in mathematical reasoning could at most approximate a probability equal to 1, as it were. In this best possible state of cognition, we could only succeed in establishing external relations among ideas that would still remain separable from one another. No matter how much we could progressively increase our mathematical reasoning abilities, we would never be entitled to reach the full certainty accorded to demonstrative knowledge, for our actual mathematical reasoning consists always in an empirical causal process. This is the same reasoning that in common life and science we regard as demonstrative knowledge after we have undergone the process of increasing our confidence in our reasoning abilities. However, we increase our confidence only because, in these contexts, we do not fully acknowledge that this increased

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[22] At T 1.4.1.3 / SBN p. 181, Hume seems very briefly to raise the possibility of a radical skeptical doubt about intuitive knowledge itself. This would go against Locke and Descartes (at least the Descartes of the *Fifth Mediation* and the *Replies* to the objections of circularity). After reminding us that knowledge and probability "cannot well run insensibly into each other," Hume writes: "Besides, if any single addition were certain, every one wou'd be so, and consequently the whole or total sum; unless the whole can be different from all its parts. I had almost said, that this was certain; but I reflect, that it must reduce *itself*, as well as every other reasoning, and from knowledge degenerate into probability." What reduces to probability here is a second level conditional about our certainty concerning the relationship between a whole (a total sum) and its parts — a conditional that seems, on the surface, to be apprehensible in a single intuitive act. However, Hume calls it "reasoning," presumably because it is a generalization of what we know about particular wholes and their parts. Thus, this suggests that Hume does not, after all, raise skeptical doubts about intuitive knowledge. A detailed analysis of the issues raised by this brief text must be left for another occasion.

assurance relies solely on probable reasoning. Hume's philosophical skeptical doubt about demonstrative knowledge is not the kind of doubt a mathematician, a merchant, or an accountant would raise precisely because, in common life and science, we do not regard the application of rules of demonstrative reasoning — in the generation of proofs — as causal processes. Thus, because we do not entertain the possibility that we might always be stuck with merely probable conclusions (no matter how high), we retain the hope that an ever-increasing improvement in the exercise of our reasoning faculties marks out the path to full certainty.[23]

As we have seen, at *Treatise* 1.3.1-2 Hume has offered an initial division between "knowledge" and "probability," following Locke's terminology but giving his own original characterization. At *Treatise* 1.3.11.2, Hume further distinguishes (explicitly in contrast with Locke) between the certainty of knowledge and two types of conviction within the second

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[23] Hume completes his radical skeptical consideration of reasoning by offering a second skeptical argument, which now concerns causal reasoning itself. In this way, in my view, Hume goes on to raise radical skeptical doubts about our attempts causally to assess and improve our causal reasoning performances. The conclusion of this second argument is that the probability of performing correct inferences gradually reduces to zero. There is an initial lack of certainty (mere probability) due to the possibility of making mistakes in a process of reasoning about, for example, a mathematical inference. But we estimate the probability of being right (or of making mistakes) by reflecting on the past successes of our faculty of reasoning, intersubjective agreement, and so on. This latter estimation is itself a judgment performed by our faculty of (probable) reasoning. Therefore, we need now to estimate the probability that the latter estimation is correct, and this new estimation further decreases the previous probability. The new estimation is in turn decreased by the same process, and so on *ad infinitum*. Hume's explicit assumption is that any finite object that is repeatedly decreased *ad infinitum* reduces to zero. Mathematically speaking this is incorrect, since, for example, we can start with the number 1 (a finite object) and generate a sequence that does not converge to zero: 1 - 1/4 - 1/8 - 1/16 - 1/32 …, and so on. This second argument has attracted more attention than the first (on which I have concentrated here), perhaps precisely because of this (apparent) mathematical blunder. [See the criticism of Hume's views on infinite divisibility by Anthony Flew, "Infinite Divisibility in Hume's *Treatise*" in *Rivista Critica di Storia della Filosofia*, 4 (1967) and the related criticism of Hume's second skeptical argument at *Treatise* 1.4.1 by Robert Fogelin, *Hume's Skepticism in the Treatise of Human Nature*, Chapter II). In my view, Hume makes this mathematically wrong assumption because, as in the discussion of infinite divisibility at *Treatise* 1.2, Hume is not concerned with objective mathematical truths (the mathematical nature of the continuum or the objective properties of probability), but rather with our ideas about them — with how they can be presented to us ostensively. According to Hume, our simple ideas — which are always copies of simple sensory impressions — must at some point vanish if we keep reducing their size, since we must eventually arrive at minima sensibilia.

kind of philosophical relations: (inductive) proofs and (mere) probability. Hume here explicitly retains the notion of certain knowledge but points out that there are cases of "probable" reasoning (as they were called in *Treatise* 1.3.2) in which we can be free from doubt and uncertainty (T 1.3.11.2 / SBN p. 124): "By knowledge, I mean the assurance arising from the comparison of ideas. By proofs, those arguments which are deriv'd from the relation of cause and effect, and which are entirely free from doubt and uncertainty." However, in inductive proofs we can be free from doubt and uncertainty only to the degree, however high, that one can be in that state of conviction concerning external relations among ideas, impressions, or objects.[24] Even in common life and science we regard the certainty of inductive proofs based on uniform experience as always open to revision, in contrast to the full certainty of intuitive and demonstrative knowledge.

In "A Letter from a Gentleman to his Friend in Edinburgh,"[25] Hume distinguishes "Kinds of Evidence into *intuitive*, *demonstrative*, *sensible*, and *moral* … *Moral* Certainty may reach as *high* a Degree of Assurance as *Mathematical*; and our Senses are surely to be comprised amongst the clearest and most convincing of all Evidences. Now, it being the Author's Purpose … to examine the Grounds of that Proposition [*That whatever begins to exist must have a Cause of Existence*]; he used the Freedom of disputing the common Opinion, that it was founded on *demonstrative* or *intuitive Certainty*; but asserts, that it is supported by *moral Evidence*,

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[24] In my paper "Hume and Locke on Scientific Methodology: The Newtonian Legacy" I argue that Hume's distinction between (inductive) proof and (mere) probability plays a crucial role in Hume's scientific methodology, and is Newtonian in character. I first pointed out that Hume's distinction between proof and (mere) probability is a reflection of Hume's Newtonianism, and, as such, marks a clear distinction with Locke, in my presentation at a symposium on Hume's remark: "A wise man, therefore, proportions his belief to the evidence" (*Enquiry*, Section X), at the Twenty-eighth Annual Hume Society Conference, Victoria, Canada, 2001. I subsequently developed the parallel with Newton and the contrast with Locke in earlier versions of "Hume and Locke on Scientific Methodology" presented at the California Institute of Technology, April 2003; the Central Division Meetings of the American Philosophical Association, April 2004 (where, in particular, I first pointed out that Hume's distinction between proof and mere probability is precisely modelled on Newton's distinction between inductive proof and conjectures or hypotheses); and the Thirty-first Annual Hume Society Conference, Tokyo, Japan, August 2004.

[25] This letter is of May 8th 1745. The following quotation is from David Hume, *An Enquiry Concerning Human Understanding. A Letter from a Gentleman to His Friend in Edinburgh*, Eric Steinberg (ed.), (Indianapolis: Hackett Publishing Company, 1977), p. 118.

and is followed by a Conviction of the same Kind with these Truths, *That all Men must die*, and that *the Sun will rise To-morrow*." This pregnant passage shows that Hume regards the "senses" (what is sensibly given before the mind) as providing ultimate evidence on a par with intuitive knowledge. It also shows that "moral certainty" is the highest degree of assurance in matters of fact based on (causal) inference. Moral certainty pertains to what at T 1.3.11.2 / SBN p. 124 Hume calls "proofs," which are no other than the best of our inductive inferences based on completely uniform experience and the principle of the uniformity of nature.

On the one hand, the examples of morally certain propositions Hume gives in "A Letter from a Gentleman" are exactly the same propositions for which we have inductive proofs according to T 1.3.11.2 / SBN p. 124: "One wou'd appear ridiculous, who wou'd say, that 'tis only [merely] probable the sun will rise to-morrow, or that all men must dye; tho' 'tis plain we have no further assurance of these facts, than what [completely uniform] experience affords us." On the other hand, at the end of *Treatise* 1.3.3, after he has argued that the maxim "every event must have a cause" is neither intuitively nor demonstratively certain, Hume indicates that he shall search for an evidential grounding of this maxim precisely by examining our inductive proofs — he wants to know (T 1.3.3.9 / SBN p. 82): "*Why we conclude, that such particular causes must necessarily have such particular effects, and why we form an inference from one to another*?" In the passages in "A Letter from a Gentleman" following the one just quoted, Hume emphasizes that the "Species of Evidence" he has enumerated must be kept distinct. Similarly, at *Treatise* 1.4.1, he reminds us that "knowledge [intuitive or demonstrative] and probability [inductive proof or "mere" probability] are of such contrary and disagreeing natures, that they cannot well run insensibly into each other." Nevertheless, Hume retains this distinction only to show us by the first skeptical argument of *Treatise* 1.4.1 that the ideal of full and entire certainty of demonstrative knowledge to which we subscribe in common life and science is actually unattainable.