

ON THE ADOPTION PROBLEM AND META-LOGICAL MONISM

Sobre el problema de la adopción y el monismo meta-lógico*

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Abstract

According to the Adoption Problem (AP) certain basic logical principles cannot be adopted. Drawing on the AP, Suki Finn presents an argument against logical pluralism: Modus Ponens (MP) and Universal Instantiation (UI) both govern a general structure shared by every logical rule. As such, analogues of these two rules must be present in every meta-logic for any logical system L, effectively imposing a restriction to logical pluralism at the meta-level through their presence constituting a “meta-logical monism”. We find a tension in the dual role that the “unadoptable rules” must play in Finn’s “meta-logical monism” rendering it ineffective to restrict logical theories and systems. Consequently, we argue they cannot be both analogues

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of MP and UI and inferentially productive. We conclude with a series of suggestions regarding where a more satisfying and robust interpretation of the AP could lie.

Key words: Rule-Following; Philosophy of Logic; Philosophy of Language; Theories of Reasoning.

Resumen

De acuerdo con el problema de la adopción (PA) ciertos principios lógicos básicos no pueden ser adoptados. A partir del PA, Suki Finn presenta un argumento contra el pluralismo lógico: tanto Modus Ponens (MP) como Instanciación Universal (IU) gobiernan una estructura general compartida por toda regla lógica. En tanto tales, versiones análogas de estas dos reglas deben estar presentes en toda meta-lógica para cualquier sistema lógico L. De esta manera, mediante su presencia, se impone una restricción al pluralismo lógico constituyendo un “monismo meta-lógico”. Identificamos una tensión en el doble papel que las “reglas inadoptables” deben jugar en el “monismo meta-lógico” de Finn que lo vuelve ineficaz para restringir teorías y sistemas lógicos. Consecuentemente, sostenemos que estas reglas no pueden ser simultáneamente versiones análogas de MP e IU e inferencialmente productivas. Concluimos con una serie de sugerencias sobre cuál podría ser una interpretación más satisfactoria y sólida del PA.

Palabras clave: Seguimiento de reglas; Filosofía de la Lógica; Filosofía del Lenguaje; Teorías del razonamiento.

1. Introduction

The Adoption Problem, as presented by Romina Padró (2021, 2015), drawing on considerations from Saul Kripke (2021), consists in showing the impossibility of adopting certain basic logical principles (or rules) by someone who was, previously to the adoption process, unable to reason in accordance with them. In order to present the problem, we are asked to imagine an agent, Harry, someone who has never inferred according to Universal Instantiation (UI) or Modus Ponens (MP) before. If we want Harry to adopt those principles, what we want, following Padró, is for Harry to begin inferring in accordance with IU or MP “on the basis of the acceptance of the corresponding logical principle” (2015, p. 31). Padró notes that “on the basis of” must not be understood in a purely causal way: what we are after is for the principle to guide Harry (2015, p. 42). Harry, even if he were to accept UI as a logical principle:

(UI) Every universal statement implies each of its instances.

will not be able to reason in accordance with it through the mere fact of having accepted it. If we want him to recognize a particular universal instantiation as a case of Universal Instantiation, in the sense of bringing the inferential pattern into practice, he must be able to reason according with UI already. But, by stipulation, that is something he could not do (Padró, 2015, p. 34). Summing up, in words of Padró:

Carroll-Kripke[-Padró] Problem (AP): certain basic logical principles cannot be adopted because, if a subject already infers in accordance with them, no adoption is needed, and if the subject does not infer in accordance with them, no adoption is possible. (2015, pp. 41-42)

Following Padró's characterization of the problem according to which some logical principles cannot be adopted, Suki Finn (2019a) sets out to explain why some basic logical principles cannot be adopted. According to her, all logical principles or rules of inference¹ share a General Structure (GS):

(GS) If the premises are an instance of structure X, then infer conclusion Y.

According to Finn, the General Structure manifests a structure which is general (or universal) because it applies in every case of type X, and conditional, because it states how to deal with every case of type X. Every rule of inference exemplifies the GS—regardless of how one decides to state it or characterize it. However, in accordance with the GS, Finn argues that the rules, MP and UI, are required to apply every describable rule following the structure specified by the GS, including MP and UI.

A rule that must be applied in a case governs that case. It is reasonable to suppose, then, that given the abovementioned, any application of a rule—which is, for the mere fact of being a rule, of the GS—presupposes the application of MP and UI. Therefore, MP and UI govern every application of a rule, including themselves. Take the Rule of Adjunction (AD):

¹ Following Padró and Kripke, who have shown how both principles or rules of inference are equally impotent (in their propositional versions, if there are others) to solve their qualms about adoption, we will speak of rules and principles as indistinct in the present context.

(AD) From a pair of propositions, their conjunction follows.

AD is an instance of the GS as it applies only to sets of premises whose structure X is (or contains) a pair of propositions. In order to apply AD, it is necessary to recognize, on the one hand, that one is faced with an instance of the rule—that is, a pair of propositions; and, on the other, that this is a case in which it is mandatory or permissible to infer the conjunction of those propositions. For the first condition one must do something resembling UI, and for the second, something like MP.

Given that rules exhibit the GS, those rules that state how to deal with universal or conditional structures will be of the structure which they govern. Since MP and UI are rules of inference themselves, they are “self-governing”. Due to this fact, self-governing rules—only UI and MP, according to Finn—turn out to be unadoptable. It is self-governance that entails the impossibility of adoption.

Finn’s reading of the Adoption Problem is interesting in that it offers concrete consequences from accepting AP’s conclusions. In a clear and succinct way, Finn manages to present what would, if correct, be a worrisome result for logicians interested in the prospects of logical pluralism. Finn’s proposal of a meta-logic M that includes at least two meta-rules (MP and UI) as a necessary aspect which Harry lacks, offers an account of what the AP only stresses in a negative fashion — specifying what cannot get an inferential practice going. As attractive as it is, in the rest of this paper we will put to the test Finn’s reading of Padró’s rendering of the AP. Our first aim is to analyze whether it can surmount a few challenges posed directly at the prospects of specifying a necessary partial monism at the meta-level M that can do the job of limiting logical pluralism. Nevertheless, the main goal of this article is not to present a defense of logical pluralism or a correction of Finn’s misunderstanding of it. What we are after is a more robust and satisfying understanding of the AP against Finn’s reconstruction—on which her attack on logical pluralism stands. We believe the temptation to read the AP as establishing the foundation over which Logic stands is strong and reasons must be given to undercut it.

The structure of the paper goes as follows: in Section 2, we offer a brief presentation of the target of Finn’s arguments, Carnap’s Principle of Tolerance, and her specific arguments against it; in the process, we present our own interpretation of Finn’s proposal, its demands, and some theoretical commitments inherited from her acceptance of the AP (sec. 2.1). In 2.3, after bridging with a closer look into Finn’s reading in 2.2, we present a discussion of a pair of concepts which are absent but

assumed in Finn's proposal: the distinction between *logica docens* and *logica utens*. This will allow us, in sec. 3, to put to the test whether Finn's construal of a partial monism at the meta-level M can accommodate the restrictions imposed by its own theoretical commitments. We provide a series of counterexamples that undermine this possibility and cast doubt on the overall interpretation of the AP Finn presents. Furthermore, we argue against the suitability of understanding that which can account for our abilities to infer as a logic, in the context of the AP. In sec. 4, we finish with a diagnosis of what we take the AP to entail against the background of a family of approaches about what logic is, that of Logical Expressivism, which shows a better prospect of stressing some of the AP's most important points, without encountering Finn's reading limitations.

2. Finn's Argument against Logical Pluralism

According to Finn, MP and UI—as “self-governing” basic inferential rules—impose a limit to logical pluralism. To understand this point, a brief explanation of logical pluralism is in order.

A logical pluralist is committed with the existence of a multiplicity of formal systems that can (or could) correctly be called “logics”. Logical pluralists deny logical monism. A logical monist claims that there is one and only one formal system that can correctly be called “logic”. The clash between pluralists and monists is approachable from different theoretical outlooks. There is no agreement—neither among pluralists nor monists—regarding what makes a system a proper logical system.

Occasionally, the labels “pluralist” and “monist” are applied to logicians whose theses could be associated, in some way, with either of these positions—broadly construed—even if they have not engaged explicitly in the discussion regarding pluralism or monism. It is in this sense that Finn takes Rudolph Carnap as committed with—or an exemplar of—a pluralist position. Carnap (1937/2002) held that, in the context of the construction of a scientific theory, there are no restrictions according to which one could present a “logic”. Rather, it is sufficient to establish the truth or use conditions which determine the meaning of the logical vocabulary of the theory to provide a formal mechanism that allows to perform inferences with the theory:

we have in every respect complete liberty with regard to the forms of language; that both the forms of construction for sentences and the rules of transformations [...] may be chosen quite arbitrarily. (Carnap, 1937/2002, p. xv)

This liberty regarding the construction of a formal language is epitomized in his so called “Principle of Tolerance” (PoT):

It is not our business to set up prohibitions, but to arrive at conventions. (Carnap, 1937/2002, p. 51)

In logic, there are no morals. Everyone is at liberty to build up his own logic, i.e., his own form of language, as he wishes. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical arguments. (p. 52)

The focus of Carnap’s PoT lies in the elaboration of clear and precise languages with (scientific) theoretical aims. It served as a methodological guideline and not as a criterion of logical adequacy. According to Finn’s interpretation, the possibility of conventionally (and hypothetically) constructing the logical terms of a system entails that, in principle, any formal system so construed can be considered a legitimate logical system. Finn argues that to make sense of his tolerance towards different systems it is necessary, beforehand, to suppose that these systems are available in the first place independently.

Finn, then, attacks the illusory appearance of freedom surrounding the PoT through her argument. In her argument, there is a meta-logical restriction to any system for it to count as a logic: it has as part of its meta-logic analogues of the self-governing rules MP and UI. What is claimed is that to be able to reason about anything, including the construction and application of a formal system integrated in a scientific framework, it is necessary to have a logic to reason with it beforehand, one that necessarily has MP and UI. Finn calls this kind of logic a “meta-logic” because it is used to reason with any system, including a logical system. Finn, consequently, intends to show that there is a limit to our theorizing of that meta-logic level M. Our logical theories must consider that they all presuppose a certain kind of monism at this meta-level M. This (partial) monism is constituted by the inescapability of MP and UI. This meta-logical partial monism does not, *prima facie*, limit “lower” logical systems L governed by this meta-level, according to Finn.

Finn’s meta-level M is said to have, in a sense we will dwell on later, the meta-logical rules or principles that allow someone to apply logical rules or principles in any system L. Finn only explicitly commits with the “partial meta-logical monism” mentioned above: the

necessity that every meta-logic M includes analogues of MP and UI. Finn's argument boils down, therefore, to defending that certain rules or principles are unavoidable in every M —the meta-level with which we reason about and with every logical system L . This imposes a limit to the logical pluralism she takes to be represented by PoT.

2.1. Interpreting Finn's argument

According to Finn, what the AP implies—due to the self-governance of basic inferential rules—is that there is a meta-logical monism consisting in that every legitimate meta-logic, every meta-logic M that can set in motion any logical system L , must have “analogues” in the meta-logic of MP and UI.

These analogues must differ in one important sense with the principles or rules of inference in the logical level L —the one we use to specify them since at the meta-level we have analogues of them—in that they must be applicable without assuming an ulterior meta-analogue of those rules at an ulterior meta-meta-level MM . This means that they must not be inferentially inert like their object level counterparts. We will call this the “Inferential Productivity Requirement” of the meta-rules (or principles).

At the same time, these analogues must coincide in sufficiently many ways with the object rules or principles of which they are said to be analogues: the inferential principles or rules at a level L , such as classical logic. We will call this the “Analogousness Requirement”. These analogues of rules must coincide in some non-trivial way with the rules we use to identify them with. At their meta-level they must function as their inferentially inert models do. It isn't clear how much these two types of rules have to coincide after setting aside their difference regarding inferential productivity. We must assume at this point that there are important analogies between meta-logical versions of object level MP and UI that justify the partial identification between the two. We will leave to the rest of the discussion further specifications of the Analogousness Requirement as required.

It is worth remembering that while the AP only demands that non-Harrys be practically able to perform inferential transitions from A to B that can be considered according to MP and UI, Finn needs analogues of MP and UI at the meta-level Harry does not have (but non-Harrys like us do).

Summing up, Finn's proposal would amount to the following: Given that the Adoption problem is true, it demands an explanation.

The AP can be explained by the property of self-governance of MP and UI which is a consequence of their governance over every application of a rule of a general and conditional structure. Therefore, given that MP and UI govern every application of a logical rule of the General Structure (GS), every logic system L needs a meta-logic M with analogues of MP and UI necessary to apply the object level rules or principles (including MP and UI).

What Finn's claims imply is that while logicians can devise any formal system L that they so desire, regardless of whether those systems have MP or UI at the object level or not, it must have analogues of MP and UI at the meta-level—regardless of the logician's awareness of that fact. An application of a rule in any system without an implicit application of inferentially productive analogues of MP and UI is impossible. Therefore, any (semantic) disagreement regarding which rules are valid presupposes a use of MP and UI—a sort of “pragmatic” agreement—an agreement in inferential performance. So, denying the validity of MP or UI would amount to a rather specific kind of performative contradiction. This, of course, differs with the usual talk of “meta-logics” in that Finn is not stating that every logical system L must have a meta-logic, on which meta-theorems are proven about it, that has MP and UI. The meta-logic in which philosophers of logic prove whether some properties hold about a system would be as inert as a logical system in Finn's sense.

2.2. A closer look to Finn's restriction to logical pluralism

At this point, we must remember Finn's opening statements. She assumes the AP to be in order and goes on to try to explain why some basic rules are unadoptable. The reason is the self-governance of MP and UI (and only them). According to Padró (2015, p. 208), one entailment of the AP is that intellectualism is false. The AP comes in the aid of Ryle against the likes of Williamson and Stanley—the contemporary champions of intellectualism.

Intellectualism: Every case of intelligent action implies a case of propositional knowledge.

Sophisticated Intellectualism (Stanley, 2011b, 2011a; Williamson & Stanley, 2001): knowing how to do something =_{equiv} knowing that there is a way to do something.

Taking the AP as true arguably implies a rejection of intellectualism in its classical “legend” formulation and its contemporary sophisti-

cated version; and takes *knowing how* as conceptually prior to *knowing that*. We cannot have *knowledge that* all the way down, at least in the case of basic logical rules—since inference is presupposed by the application of a rule (Padró, 2015, p. 208). In Padró's words:

The adoption problem calls into question the view that rules guide our basic deductive inferential performances, and with it the relevance of propositional content in the context of an inferential act. (2015, p. 207)

So, propositional knowledge is insufficient for an intelligent performance. What is missing is knowledge when and how such propositional knowledge should be applied for the performance to be successful. (2015, p. 200)

Finn appears to agree on this characterization of what Harry is missing; she claims that Harry's "novice" status deprives him of knowing how or when to apply the rule:

Harry's "novice" status paralyzes him, since without the corresponding inferential practice in place (or as I take it, UI in the meta-logic), Harry cannot know how or when to apply the rule. (Finn, 2019a, p. 5)

UI and MP are self-governing in that all logical rules are of conditional and universal structure, such that they unavoidably presuppose an understanding of UI and MP. Therefore, all logical rules require one to already know how and when to apply UI and MP, and this is what makes UI and MP basic, since all other logical rules presuppose them. (Finn, 2019a, p. 15)

In another paper dealing with the AP and its consequences for Carnap's (and Quine's) philosophy of logic, she continues using the same vocabulary regarding what is needed to follow rules, linking it to Wright's arguments regarding Rule-Circularity (Wright, 1989):

Related to the AP is that, to follow any rule successfully requires two steps, first to know when the rule applies, and second to know how to apply it. The first step is to identify a structure and the appropriate rule for that structure, and the second step is to use that rule to infer a conclusion. (Finn, 2019b, p. 244)

As such, in what follows we will consider Finn as committed with the rejection of intellectualism. A rule novice in Finn's sense is

someone that lacks knowledge of how and when, and not only (if at all) knowledge that. This *know how* the novice lacks must be understood against the backdrop of Ryle's (1945, 1949/2009) classic arguments against intellectualism. Knowing how to apply basic inferential rules is conceptually prior and independent of knowledge that (explicit propositional knowledge). In Padró's terms, the most basic aspects of what is sometimes called *logica utens*—logic in use or “in possession”—is what the novice (Harry) lacks.

2.3. *Logica docens and logica utens*

The main way in which the anti-intellectualist point of the AP is visible throughout Padró's presentation of the problem is through the distinction between two different *kinds* of logic: the medieval distinction, revived mainly by Peirce, between *logica docens* and *logica utens*.

Logica docens is logical theory—logic as taught. As such, it is a kind of knowing that. One knows a *logica docens* propositionally. Contrary to *logica docens* we have *logica utens*—logic in possession. It is the logic we *use* pre-theoretically. According to Peirce, it is the logic we use while reasoning. Reasoning is not to be confused with inferring. Inferring, then, is a “compulsive instinct or habit” (CP, 2.204)², and reasoning is a willful act of *intending* to follow a principle of reasoning. *Logica docens*, as a theory, stands normatively to inferring as something to correct what we do in reasoning, inferentially speaking. It is a theory about logical principles, not about actual inferential behavior. In this sense, it is reminiscent of the classic Fregean arguments against psychologism (CP, 2.204, 2.27). According to Peirce, neither *logica docens* nor *utens* is a matter of the psychology of inference. Both, in different ways, stand as goals of successful or correct reasoning to the actual inferring behavior. *Logica utens* does it informally or pre-theoretically; it is what we call what an agent intends to *do* inferentially. *Logica docens* is what the logician says is a correct system of reasoning or logical consequence—vaguely put. What matters is that while *logica docens* is propositional logical knowledge if anything is, *logica utens* is not as clear cut—even in Peirce's rendering of the distinction. Padró explicitly takes *logica utens* as equivalent to logical practice or inferring behavior. At least, from the standpoint of the AP, inferring behavior and “reasoning” are not bound by Peirce's strictures:

² CP refers to the notation of *The Collected Papers of Charles Sanders Peirce* (1994).

[...] what Harry lacks is the ‘logic in possession,’ the *logica utens*. He is unable to perform inferences that, from the *logica docens* point of view, we would classify as instances of the UI pattern. Our attempt to remedy his inferential gap comes, on the other hand, from the *logica docens*. The adoption problem can be seen as a way of bringing out the issue of the connection between the *logica docens* and the *logica utens*: it challenges the idea that the *logica utens* (at least at a very basic level) requires the (implicit or explicit) acceptance of logical principles or rules (*logica docens*). (Padró, 2015, p. 47)

Padró, then, takes the AP as grounds to reject Peirce’s conception of *logica utens* as too restrictive. According to her, accepting it would entail that we would only be properly reasoning when we were consciously trying to follow principles or rules (Padró, 2015, p. 46). On Padró’s version of *logica utens* inferring and reasoning do not need to be distinguished. Where Peirce would force us to take inferring as a case of “conforming to a rule” and reasoning of “following a rule”, Padró chooses to call what she is after as “proceeding according to rules”, which is left ambiguous between the two. A *logica utens* in the case of Padró includes an account of inferring and in Peirce’s case it does not. And this is no surprise because what Harry lacks is the ability to infer, something that Padró sometimes details as him lacking a *logica utens*. In sum, as Padró’s rendering of the AP states, one can have a *logica utens* (as the ability to infer) without having a *logica docens*—any logical principle of a propositional form whatsoever—but not the opposite: *Logica docens*—the grasp of logical principles—presupposes a *logica utens*—the ability to infer according to them.

With the distinction between *logica utens* and *docens* in mind, and before considering which sense of a meta-logic M Finn could be said to be using, we should consider what the systems L that would be restricted by that meta-logic M are: these are either a *logica utens* or a *logica docens*. Considering that Carnap’s candidates to be tolerant about are propositional and conventional theories, the level of logical systems L that Finn takes to be constrained by the monism of her proposed meta-logic must be taken to be a kind of *logica docens*. Having seen that it is *logica docens* candidates (explicit logical theories) that are restricted, we can consider what kind of logic a meta-logic M that must involve unadoptable rules could be.

So, the meta-logical level M with unadoptable rules is either a *logica docens* or a *logica utens*. Some logics should not need a meta-logical level for being applied, as some logics *must* be able to apply those

logics that cannot be applied by themselves without needing another logic to apply them, at the risk of an infinite regress of logics. No *logica docens* would be a good candidate for applying itself. Logical principles are applied, not applicers—in the relevant sense. A *logica docens* fails the Inferential Productivity Requirement: *docens* systems L are propositional and, according to Finn herself (following Kripke, Padró, and Priest), inferentially inert (Finn, 2019a, p. 18; Kripke, 2021; Padró, 2015, p. 194).

What we mean by this is that formal systems (being propositional and explicit) cannot apply themselves but presuppose a meta-logical level that applies them. A geometry does not require a meta-geometry, it requires a logic, but a logic requires a meta-logic to apply it—and this cannot go on forever (unlike ordinary semantic ascent). Since what Finn needs cannot be, itself, a *logica docens*, we must be dealing either with a *logica utens*, or something else entirely. If Harry lacks a *logica utens*—like Padró claims, then, what he needs is something to grant him individual basic inferring behavior. Something that would specify his *know-how*. Finn cannot use other renderings of what a *logica utens* is if she were to claim that what Harry lacks is, after all, a *logica utens*—the knowledge of how and when to apply the rule. Therefore, Finn could be understood as taking MP and UI as rules or principles that must be included into any and every meta-level M understood as a *logica utens* at the individual level—as part of the inferential *know-how* of an agent, not propositional knowledge. Whether it is also shared is unimportant.

This pushes a dual constraint on the range of possibilities Finn has for her monist meta-logic understood as a *logica utens* in the sense of what explains inferences. But Finn also requires something the AP as presented by Padró does not: counterparts of logic level L rules or principles worthy of being so called while differing in their inferential inertness at the meta-level. That is, Finn's meta-rules in a *logica utens* sense must satisfy both i) the Inferential Productivity Requirement and ii) the Analogousness Requirement. The AP only concludes that one cannot get an inferentially productive practice out of mere acceptance of logical rules or principles. We must have performed inferences according to basic inferential rules like UI before being able to follow them. While the AP states a negative point, Finn produces a positive account of a necessary condition for getting an inferential practice going. It is this last aspect that will be put to the test in what follows.

Finn does not present her own alternative as to what kind of thing Harry lacks, other than her description of the monist meta-logic and that it involves or *is* knowing when and how to apply rules. According to Padró (2015, ch. 7), the most salient alternatives are i) habits or

dispositions, ii) learnable abilities or skills (if these are regarded as something apart from habits or dispositions in the first place), or iii) “hardwired” capacities, abilities or skills. Things like these come to the fore because they are meant to fulfill the Inferential Productivity Requirement.

These dispositions or skills (learnable or not), in turn, can only be described from a propositional standpoint. So, it makes sense to say something like “John has the disposition to infer in accordance with Modus Ponens”, taking this disposition to be the practical ability specified as a resulting successful performance of it. To infer in accordance with MP should not be confused with having MP itself as a logical rule or principle capable of its own application in this setting as that would entail a kind of intellectualism, which would be at odds with the AP.

In the next section we consider whether a *logica utens* in this sense can be what Finn needs it to be, and whether if it restricts, in the way she claims, the prospects of either logical pluralism or what can be legitimately called a logic at all.

3. Challenging Finn’s Meta-Logical Rules

If Finn is saying that Harry lacks MP and UI as part of a *logica utens* and MP and UI are analogues of *rules* of inference, then we must make sure that we are clear on what we understand as MP and UI at the meta-level M Finn is talking about. Otherwise, the restriction to logical pluralism could fall into a mere equivocation. If MP and UI at the meta-level must really be in every meta-logic M needed for applying every inference rule, then their content must be precisely stated—even after allowing for their difference in inferential productivity.

Here we intend to show that other descriptions of regular inferential behavior (to put it as neutrally as possible), taken to be inferentially productive, could do the job Harry cannot do, without being interesting analogues of MP or UI. Finn’s meta-logic must specify a kind of know-how. The know-how Harry simply does not have.

What is available for Finn to play as candidates for meta-rules that satisfy both requirements are, then, dispositions or abilities to perform inferences, learnable in some non-propositional way by training, perhaps, or not. Since it is irrelevant to commit either us or Finn with the *one true option* among the ones considered to account for the knowledge of when and how to apply a rule that MP and UI represent, we will just call whatever that is the “Know-How Base” (KHB) of a broadly understood capacity.

The KHB must be non-linguistic in nature—without it, language would probably be impossible, so it must be characterized by what it allows one to do: as practical functions of an agent. We can characterize the KHBs linguistically as we can describe any ability or property able to explain behavior—for instance, our capacity of riding bikes—just as what it does. A KHB can be considered as simple or complex as one’s needs are. Take, for instance, the KHB that allows one to get from point A to B: “When in point A, move in direction x until you reach B”. Surely, it would require the KHB to allow one to recognize, at least “automatically”, one is in point A and that one reached point B.

In what follows we provide a series of counterexamples that challenge the idea that only analogues of MP or UI can do the job required of them by Finn, by presenting multiple descriptions for the KHBs required to apply the principles or rules that Finn takes Harry as lacking and non-Harrys like us as possessing. We aim to show that descriptions of KHBs sufficient to get Harry out of his predicament, which interestingly cannot be called analogues of MP or UI, can be provided.

We now must face the problem of specifying what counts as an analogue of a rule of inference and what does not. Their role in Finn’s argument can provide a non-arbitrary clue to what level of similarity must be met. For the meta-rules to perform their restrictive role in Finn’s argument towards specifying which *logica docens* systems are worthy of being called *Logic*, the meta-rules must only be describable as inferential capacities worthy of the role they have in logical systems. Furthermore, since non-Harrys are claimed to have these meta-logical rules themselves, no description of the KHB that a non-Harry has must be available that could count as producing radically different inferential behavior than what the object level rule the meta-rule is identified with would produce.

3.1. Why inferential KHBs should not be identified with logical rules

Following the AP, we know that Harry cannot perform inferences that fall into this reasoning pattern:

$$\{A \rightarrow B, A\} \vdash B$$

He has never performed one and cannot perform one even under the assumption that he comes to accept MP as a logical principle (or rule):

(MP) From a conditional and its antecedent, infer the consequent.³

So far, so good. Now, let us assume a non-Harry, someone who must, as far as the argument goes, be able to reason according to MP. This non-Harry, let us call her Mary, infers in a way that can be described as an instance of the following reasoning pattern:

$$\{(A \wedge B) \rightarrow B, (A \wedge B)\} \vdash B$$

It contains two premises, one of which is a complex conditional, the other stating its antecedent. The rule of Modus Ponens (MP) could thereby be said to govern it in Finn's terms, insofar the premises of the reasoning have the adequate structure for MP. It conforms with MP, in short.

However, the reasoning is in conformity with other imaginable courses of inferential action as well. Take the rule MP* as an example:

(MP*) Given a conditional with a complex antecedent and its antecedent, infer its consequent.

If an agent reasoned in this way, she would reason in accordance with MP, but then also in accordance with MP*. MP* is, somehow, a restricted version of MP, as it pertains to a set of premises whose structure is governed by MP, but that involves only complex antecedents of the major premise. MP governs all of its instances, like MP*. In any case, this difference makes no difference that would count as an inadequate or inadmissible description of the capacity or ability to infer Harry lacks. If an agent (say, Mary) had a KHB that made her infer in a way that accords with MP*, she would not be a Harry. She would be able to reason in accordance with MP, as both Padró and Finn claim Harry cannot, but not in all cases. She would just not infer B from $A \rightarrow B$ and A.

In any case, the structure displayed by the premises is not the only thing determining what is adequate as a description of a KHB, since other rule-like descriptions could refer to other aspects that distinguish an inferential pattern from another, such as in the following pseudo-inferential rule:

³ In this section we present our examples with MP, but the argument could easily accommodate UI for all or most examples considered.

(MP^{5K}) Given a conditional and its antecedent, before the year 5000, infer the consequent.

Once again, this characterization of inferring tendencies (a KHB) has a different extension than that of MP.

Even though there is no difference in structure, which explains the conformity of the KHB with MP in a set of cases (all of which are performed before the 5000's), there will be a point in which the ability, up to then in accordance with MP, will no longer be describable as conforming to it. If Mary had MP^{5K} as her KHB (that is, she would not be a novice), she would not need to adopt MP since she would be able to infer according to MP, without having an interesting analogue of MP (something that could be called a *logical* rule), in enough cases. But maybe, it could be argued, this ability to infer according to MP^{5K} really does fulfill the Analogousness Requirement. After all, there would be no difference in inferential behavior to note before this extremely long time. But if Mary did have MP^{5K} as a KHB allowing her to infer according to our description of it, would we be prepared to call *that* a meta-rule that logical pluralists would need to respect or cannot avoid having in their (partially) monist meta-logic? Surely something has gone amiss. Mary, after all, having the KHB to produce MP^{5K} inferences, can infer in ways that Harry could never dream of for over two thousand years. She can apply whatever principles or rules we so desire for that time; so, what could be said against it from Finn's standpoint? In our lifetimes as non-Harrys, we will get to perform far fewer inferences. We cannot seem to feel pity for her without feeling sorrier for ourselves.

What allows the underdetermination of the KHBs required to perform inferences governed by the same logical rules is that, to press an old point, these govern over an infinity of cases, while the actual inferences produced with those KHBs will always be finite and, arguably, inherently fallible. But Finn does not present an argument that would allow her meta-rules (constituted by KHBs) to match, in extension and properties, their lower-level models like MP or UI as part of a *logica docens* to block these kinds of alternative KHBs, and it is difficult to imagine how that argument could run.

The offered KHBs capable of producing different inferential patterns are very similar to MP in many contexts. But the offered examples underdetermine the identification of that which Harry lacks with what Finn requires for her purposes. Which rule one takes an inferential transition to be governed by is, at some level, a matter of stipulation. It is not enough to say that every rule involves something

general and something conditional to say that it is governed by MP or UI—to constitute a KHB identified with every KHB producing inferences governed by MP or UI—, because the range of generality and the kind of conditionality involved can vary a great deal.

At the level of generality Finn is talking about, the analogues of MP or UI that would be needed to apply rules could be something very different from MP and UI. And saying “whatever one is using to draw inferences is actually MP or UI” is of little help too. A practical equivalence class of KHBs is enough, in our reading. What Finn needs is an argument showing that only through *precise* analogues of MP and UI can someone do the job she asks of them. Another option would be a minimum, non-arbitrary, threshold for KHBs to pass that allows one to apply any rule.

We will stop here, considering the examples given are sufficient to show that, given a set of inferences that are supposed to be governed by MP and UI and that partially conform with MP (or equivalent examples using UI), it is possible to offer alternative descriptions for what is needed to perform them, different KHBs—without rejecting the conclusion of the AP. Moreover, without a positive argument in favor of a criterion for choosing among them that would privilege MP or UI (even accounting for the finite-infinite difference between KHBs and rules of inference), it is not possible to pick one as the only correct description of inferential abilities, independent of their propositional counterparts that allegedly require them, and not the other way around. This, we argue, presents a difficulty for Finn’s attempt to identify the required KHBs for applying a rule with MP (or UI) in a non-trivial way.

It could be objected that a skeptical underdetermination of the identification of the analogues of MP and UI said to be in the meta-logic through deviant examples, like the one presented, changes the subject or amounts to a merely verbal objection. It must be stressed that we are not claiming that someone who was said to have MP and UI as rules in a meta-logic M understood as *utens* could not know which rule she was applying. That is, we are not saying that she would not be able to apply them because of that underdetermination (cf. Boghossian, 2012, p. 13). Rather, it would make no difference if she had a KHB that involves analogues of MP or UI at the meta-level or the deviant, but functional, KHBs we considered. Each proposal offered (and others like them) would allow someone to infer according to MP and UI (in Finn’s sense) without being MP and UI themselves. Someone with these KHBs would not be in Harry’s predicament, she would not be a “novice” in Finn’s sense, but, at the same time, would not have MP or UI in the sense Finn requires.

If multiple descriptions of inferential abilities or capacities could play the role Finn claims only MP and UI—as self-governing rules of inference—can play, then the partial meta-logical monism Finn presents dilutes away. It would no longer be clear what role MP and UI play, in their meta-rule form, as presupposed by any application of a rule. At most, what Finn calls “analogues of MP and UI at the meta-level” could be understood as a “practical equivalence class” of KHBs that would allow someone to apply logical rules or principles. If this is so, new questions arise, like “Why call this a logic, even a meta-logic *utens*, at all?”. In other words, why call the class of things that allows one to reason *in practice* in the way that Harry cannot a *logic*?

It should be clear by now that this class does not need to be populated by “rules” worthy of the attention of logicians. Following Finn’s own argument seems to be pointing towards whatever allows one to reason—even if it is not a logic. The only thing we appear to know about *that* is that it is not propositional knowledge, something that the AP already entails. Only assuming the univocal characterization of that which would do the trick to get us to produce inferences according to MP and UI could we privilege these principles or rules. But this seems to demand a great capacity to forget the differences of an indefinite number of viable KHBs.

In the next sub-section, we will approach the matter from another angle, arguing against KHBs as the right kind of thing to consider as a logic in the relevant sense.

3.2. *Stressing the difference between KHBs and a Logic*

As a final argument undermining the plausibility of identifying an interesting KHB (singular or plural) necessary for an agent to perform any inference on the account of rules of inference being describable as of the General Structure (see sec. 2 above), consider the following:

When heated to 30° C, dilate until you reach the mark for 30° C in the glass casing.

Mercury has the disposition to do that under certain conditions like pressure, being in a container of the appropriate kind, and the like, that is, *ceteris paribus*, or fallibly. What mercury does is not an inference, and, as such is not governed by MP, that would just be a category mistake. But what if we had a person that automatically went to grab a glass of water when the temperature reached 30° C? Someone disposed as follows:

When the temperature reaches 30° C, get a glass of water.

Her dispositions would be quite similar, as far as describing goes, compared to the ones of the mercury in the thermometer. What makes a disposition like this unable to be of the same nature as one constituting the KHB to perform an inference? Something like:

When the temperature reaches 30° C in the spring, you are allowed to say: "It's really hot for this time of the year".

She would know when and how to say that. Is that an inference worthy of being governed by MP and UI yet, or be called an inference at all? According to Wittgenstein on *Remarks on the Foundation of Mathematics*, this agent would be performing an inference, just not a logical one:

Imagine a procedure in which someone who is pushing a wheelbarrow comes to realize that he must clean the axle of the wheel when the wheelbarrow gets too difficult to push. I don't mean that he says to himself: "Whenever the wheelbarrow can't be pushed...", but he simply acts in this way. And he happens to shout to someone else: "The wheelbarrow won't push; clean the axle", or again: "The wheelbarrow won't push. So the axle needs cleaning." Now this is an inference. Not a logical inference, of course (Wittgenstein, 1981, VII-30)

If so, having the KHB necessary to perform inferences would not grant someone a logic. Maybe a parrot can do or say things like that when some temperature is reached, and even animals without sophisticated vocal gestures can realize that some actions are ripe for doing when some conditions are met like in the case of the wheelbarrow and the dirty axle. For instance, to prowl and hunt when an appropriately sized prey is in their vicinity. Surely, attributing them any logic is kind of a stretch. What about this next rule?

When in the presence of a conditional and its antecedent (if they are acceptable), state or tacitly accept (as appropriate) the consequent.

Surely the KHB necessary to know when one is in the presence of such and such is a complex matter so the KHB to produce, automatically, the conclusion of a MP from its premises is too (the KHB to know how

to perform the appropriate action). Now, is there a way to know if all agents must perform this action through the same means? If not, what do we gain by saying that we are not only able to perform basic inferences according to MP and UI, like classical Modus Ponens, but that we already have those rules in a meta-logic? Surely the KHB to perform MP or UI involves the KHB to know when *not* to perform MPs or UIs. Otherwise, we would have no means to know whether someone did know or just randomly performed something that just appears to be MP or UI.

KHBs necessary to infer competently involve other competences potentially unspecifiable but that are integral to their proper function in advance, while the specification of a logical rule does not, and the former are probably more complex than just a description of having correctly performed the inference—KHBs, as abilities or dispositions (or even future successor concepts), are rarely described other than using their success cases as paradigms.⁴ Since what we are after is a non-trivial explanation of the ability to infer or what allows for the inference itself, just describing what counts as succeeding is not of much help in these cases. If the arguments above are sound, what this suggests is that it can be fruitful to distinguish between logical principles like MP and UI and our knowing how and when to apply rules that Finn's argument purports to represent with "analogues" of MP and UI, resisting the temptation of calling them logical principles or rules in a loaded sense.

In any case, what if there are still reasons to want what Finn appears to be offering us: a neat and unavoidable Ur-logic (Kripke, 2021) or intuitive logic that really does all the job that no *logica docens* can do? What if Finn was only talking about the required competence in applying rules and calls *that* analogues of MP and UI at the meta-logic? We would do well to remember that a good amount of training is necessary long before one can take advantage of a basic logic course. Finn could be talking of just that hard earned competence that we take for granted when we arrive at our first logic class or textbook. Without it, we could not even understand what is asked of us when presented with logical rules. Understood in this way, Finn's meta-logic would represent minimum but necessary demands that different inferential communities expect of new members (Kripke, 1982, pp. 89-92). If they asked for less than skills only equivalent of "analogues" of MP or UI, we could not understand what

⁴ For short lived, and easily bored, beings like us, at least. One could have an ability (in the loose terms we think of KHBs) "described" in negative terms, "to make a hole in one in golf is not to bounce a basketball, nor to drink coffee, nor to ..., and so on."

they were doing *as* an inferential practice. Taking Kripke's reading of Wittgenstein (Kripke, 1982) as a model for what Finn is advancing, we, as inferential agents, would not need to know whether we were really using MP or not to count as having inferred according to MP. All we would need to be able to do is to perform enough MP inferences in the right way (the way of our community), and that is the only thing that can be asked of us. We have no qualms in accepting this. But is this what Finn is saying? How could rough and ready criteria for taking someone to be following rules, and not just performing inferences randomly, constitute the class of things that logicians ought to respect? Especially when a *logica docens* is required to describe them in the first place. If nothing more is provided to specify what this minimum competence is, besides that it will necessarily involve producing inferences that are of the General Structure (and, as such, governed by MP and UI) some of the time, then no interesting demarcation can be made of it. And without that, we cannot even begin to think of calling that *a* logic.

4. The AP and its Family Resemblance with Logical Expressivism

It is worth reminding the reader that the focus of this article, up to this point, was not so much a defense of logical pluralism as a series of arguments against Finn's interpretation of the AP that enables her attack on logical pluralism. As Finn's interpretation entails a series of commitments regarding what Logic is and how logical theorizing relates to our inferential actions, we set out to argue against those commitments where we found them lacking.

For the sake of making our point clearer, imagine a simplified scenario of the developing of a first *explicit* account of logic, a proto *logica docens*, if you will, out of a community of three agents in which each possesses slightly different KHBs to infer. To make things as simple as possible, we will grant ourselves the superpower to read their KHBs directly, not through their behavior. We have our *logica docens* knowledge intact and can describe their KHBs and what they will come to say about them with our logical vocabulary as well.

Now, for the members of the tribe, we first have Ann, who has the KHB to infer according to MP. She can identify inferences (correctly, as our superpower to read KHBs lets us know) that are of the MP pattern and act accordingly. When she is given a conditional and its antecedent, she is able to recognize them, *ceteris paribus*, as such and, if she accepts them, she either asserts the conclusion or tacitly endorses it when the

other members of her tribe do the same. She did not accept the MP principle (tacitly or explicitly), she just recognizes premises that we would call of an MP pattern, and when she does (even if she does not know they *are* of the MP pattern), she *sees*, intuitively—as in “quite directly”—that the conclusion is acceptable if the premises are.

A different case is that of Bob. Bob has the KHB to infer according to MP* (see sec. 3.1 above). When he accepts a conditional statement with a complex antecedent and that same antecedent as another premise, he goes on to treat the conclusion just as he treats the premises. Now, we want to know exactly why Bob cannot see that MP inferences are as good as MP*, intuitively. Well, it happens that Bob just sees the relationship that a complex antecedent has with itself as a freestanding proposition but gets a little confused with simple propositions in their place. He just does not get the conclusion to pop in his mind (or mouth) when a conditional comes before his consideration with a simple antecedent and then the latter as a freestanding asserted proposition. Bob has a blind spot in his KHBs to infer just as we had one with the inference pattern: ‘All As are Bs \vdash Some As are Bs’ before someone noticed the possibility of the subject term being empty (Kripke, 2021).

So much for Bob’s dispositions. The last member of this tribe is Charlie. Charlie has a complex disposition to perform MP inferences only when people are talking peacefully and agreeing about things, so when Ann and Bob agree on something, he is disposed to reason according to MP (like Ann), but when they don’t, he just won’t infer. He does not recognize MP inferences as in order at all during heated arguments while he would immediately treat them as acceptable during agreeable conversations.

Now, we can expect some trouble in this little community. They share enough inferential regularities to get along most of the time, but disagreements are bound to arise between Ann and Bob and, when that happens, Charlie will not be of much help. They are disposed to accept something that would get them to agree more. What they need is some good old primitive *logica docens*, or, at least, rudimentary explicit talk about inferential patterns.

How could they come to do that? Well, if Ann could get Bob to see the merits of her way of inferring, by describing it and showing him ways to get to exercise his limited KHB and extend it towards something more MP-like (closer to what Ann does), then they would be able to find more common ground than before. She could get him to follow her lead and notice that whenever he sees an MP inference without a complex antecedent in the major premise, he should just double the

antecedent down and get $A \ \& \ A$ from A . Now an MP inference will be of the MP^* pattern. It might take a little while, but Bob would likely get to see Ann's point. After their last disagreements regarding MP and MP^* vanish, they won't encounter any issues presenting the newly minted MP rule (or principle) to Charlie.

Notice that this MP proto rule does not function as a representation of Ann's KHB. It does, in a way, but that's not the reason why it would be embraced as a rudimentary logical principle. It would be used as a reminder, as an external standard to each of their KHBs. To see this last point, remember that Ann only has the KHB to infer according to MP *ceteris paribus*, as we are not bothering to detail all the circumstances that would prevent Ann from inferring according to MP, they are just too many. We can easily imagine that she had the dispositions to infer according to MP for the first 35 years of her life. On her 36th birthday, the MP rule would not lose an ounce of its original point for having ceased to represent what Ann is inclined to do. That's what would make it normative, it serves as a standard against which to compare inferential actions just as a ruler is a standard made to make measurements.

This simple tale about a primitive inferential community is actually pretty close to the kinds of genealogical explanations that expressivists offer about the nature and origins of logic. These kinds of narratives help to realign our focus and understand the role—the difference it makes to practice—of a logical concept or theory. Against Dummett, Price uses a fictional genealogy of negation to claim that even if intuitionist logic was more basic, a community that could have started to use classical negation would have been benefitted with an important upgrade in their expressive capabilities (Price, 1983, 1990, 2015; Ripley, 2011). The argument rests not on the truth of the double negation rule but on the role it plays in natural language, particularly in disagreement. It is not a matter of utility but of whether it would have a *use* by a community, without requiring explicit arguments regarding their utility. A community that reasoned classically would be able to express disagreements more successfully and would find more opportunities to address them.

Brandom (1998, 2001, 2008) does something similar with the emergence of conditionals as an expressive tool in our languages. Take Wittgenstein's "non-logical" inference regarding the dirty wheelbarrow. If two people wanted to discuss about it, a conditional would be of great help to do that. By claiming "if A, then B" one would be able to endorse, assert, the inferential transition between A and B that one, previously, would only be able to perform. Think of trying to present counterexamples

only looking at what he was doing without him ever committing to doing *that*. Nevertheless, Brandom stresses that conditionals or explicit rules could never get someone to infer if he wasn't able to do it practically (or "materially", in his terms), without recourse to logical vocabulary (like conditionals) beforehand.

Expressivism approaches branch in different explanatory projects for the philosophy of logic and carry forward commitments in the philosophy of language. Price, for instance, advances his argument on the expressive role of negation in what has come to be known as "bilateralist" or "rejectivist" approaches against preferring intuitionism for the best logic to model our natural language inferences. Brandom defends a broad inferentialist approach that takes our normative attitudes towards inferences both material and logical and our inferential capabilities as primitive.⁵

The AP, regardless of our rejection of Finn's elaboration, draws our attention to that point where we can't get any further in grounding logic, we just have to assume our inferential abilities and resist temptations to explain them *in* logic. We might have interesting things to say about what enables to infer from a psychological or sociological standpoint, but that is just engaging in another sort of inquiry.

Given our results, two options come to the fore as the most plausible additions to what the AP states that would underscore its point. Either *logica utens* starts somewhere between where Priest (2014) and Peirce say: as the production of basic norms of inference that we can take agents as using in correcting each other, or themselves; or a *logica utens* contains an assumption of our inferential capabilities understood as primitive—for logic's purpose. In an expressivist tone, we come to logical principles as hypothetical understandings of what we always *ought* to do while inferring, the best version of our understanding of those capabilities which can then be used as standards (up to a limit) to help us improve our inferential practices. They are not useless if one ceases to think them as *enabling* us to infer in the first place. They are useful only to beings that *can* infer, like us. And this, although it extends beyond the reach of this paper, could help make clear the point that only through

⁵ While exceeding the scope of this paper, Buacar (2015) offers a thorough analysis of inferential approaches, including some treatment of expressivism. Regarding what she considers the best version of inferentialism (Brandom's normative kind), she proposes an approach that could help alleviate some of the biggest problems she encounters with a complementary account of the inferential learning process (cf. Brandom, 2008), bridging the gap between inferential practice and logical theory. This program, in our opinion, is worth exploring as broadly compatible with what the AP entails.

reasoning about reasoning can we get evidence to prefer one *logica docens* or the other (Finn, 2019b; Kripke, 2021). There is no shortcut to logical theorizing, not through physical evidence—*pace* Putnam, nor through a partial description of the essence of logical rules—*pace* Finn.

In this way, on our reading, the AP should be seen as pointing towards the roots of our inferential practices in *practice*, and not to the grounding of our logical systems in a mysterious meta-logic we cannot lack. As such, the AP presents a difficult road with interesting and puzzling questions regarding inference and its relationship with rationality instead of providing sharp prohibitions for logical system building.

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