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PARACONSISTENCY, EVIDENCE AND SEMANTIC INCOMPLETENESS

Paraconsistencia, evidencia e incompletitud semántica

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Abstract

In this paper, we argue that the systems *Basic Logic of Evidence* (BLE) and *Logic of Evidence and Truth* (LET_J) suffer a kind of semantic incompleteness with respect to the informal notion of evidence. More especifically, we argue that the connective *o* of the logic LET_J fails to validate intuitive principles about conclusive evidence.

Key words: Paraconsistent Logics; Evidence; Philosophical Interpretations; Philosophy of Logic.

Resumen

En este artículo argumentamos que los sistemas *Lógica Básica de la Evidencia* (BLE) y *Lógica de la Evidencia y Verdad* (LET_J) sufren una especie de incompletud semántica con respecto a la noción informal de evidencia. Más específicamente, argumentamos que el conectivo *o* de la lógica LET_J no valida principios intuitivos sobre la evidencia concluyente.

Palabras clave: Lógicas paraconsistentes; Evidencia; Interpretaciones filosóficas; Filosofía de la lógica.

1. Introduction

The Brazilian school of paraconsistency has an important place in the epistemic approach to contradictions. According to some prominent members of this school, contradictions result from our limited cognitive apparatus, the flow of information in databases, and scientific theories.[1] Such an idea departs from the ontological interpretation of

[1] Carnielli and Rodrigues widely defend this epistemological standpoint about contradictions in some of their works. On the other hand, it is important to point out that this position about the nature of contradictions is not shared by all the

paraconsistency, which takes contradictions as reflecting something real. So, it is necessary to provide a formal apparatus that is compatible with the proposed interpretation, and the *Logics of Formal Inconsistency* (LFIs, Carnielli et al., 2007) are proposed as a basis for this approach. LFIs are paraconsistent logics that have resources to recover classical inferences. Carnielli and Rodrigues (2015) propose to interpret the minimal LFI, the logic mbC (Carnielli et al., 2007), in terms of preservation of evidence. In this logic, $\neg A$ means "there is an evidence that A is not the case". If the evidence is not conclusive, we may have that both A and $\neg A$ are the case. The problem with mbC is that it does not capture situations where we have evidence neither for A nor for $\neg A$. Later on, the authors propose in a series of papers (Carnielli & Rodrigues 2015, 2019a, b, c) two logics that are allegedly more adequate to capture the notion of evidence: the *Basic Logic of Evidence* (BLE) and the *Logic of Evidence and Truth* (LET_J), where the latter is a LFI. The logic BLE is proposed to capture the general properties of preservation of evidence and LET_J extends BLE with an operator "o" capable of recovering classical inferences. In LET_J, oA means "there is a conclusive evidence for A". As the authors argue, both logics are not intended to rival classical logic, because they deal with different things: classical logic deals with preservation of truth, while LET_J and BLE deal with preservation of evidence.

The possibility of recovering classical inferences, in LET_J, with the aid of the operator o is certainly a powerful and promising idea. Such a possibility may be taken as an advantage over many paraconsistent logics that do not have sufficient expressive power of recovering some basic classical inferences, such as contraposition. However, Carnielli and Rodrigues' proposal faces some objections in the literature. First, Barrio (2018) argues that the formal systems BLE and LET_J themselves do not commit us to an epistemic interpretation of paraconsistency. That is, there is nothing in these formal systems that prohibits an ontological interpretation of paraconsistency. Second, Lo Guercio and Szmuc (2018) argue that the formalization of evidence by means of BLE forces us to an extreme form of permissivism, which is controversial

members of the Brazilian tradition of paraconsistency. Among the paraconsistent systems proposed to deal with contradictions present in mathematics and scientific theories in general (e.g., D'Ottaviano & da Costa, 1970; Sette, 1973; da Costa, 1974; Antunes, 2018), we find applications of paraconsistent systems to dialectics (da Costa & Marconi, 1989). Da Costa (1982) argues that one of the philosophical applications of these logics is in Meinong's theory of objects. We thank one of the referees for calling our attention to this subtlety.

among epistemologists. Third, Arenhart (2021) argues that evidence does not seem to be adequately captured by paraconsistent logics. His main objection is that evidence requires a non-adjunctive approach, contrary to Carnielli and Rodrigues' approach.

Here, we will argue that the logic LET_J also suffers a certain incompleteness with respect to its intended interpretation. That is, the principles governing *o* are too general to allow us to say that it really captures conclusive evidence. This paper is structured as follows. In Section 2, we present the basic ideas of the logics BLE and LET_J, and their philosophical motivations. In Section 3, we outline the objections that Carnielli and Rodrigues face in the literature. In Section 4, we argue that LET_J is semantically incomplete with respect to its informal interpretation. In Section 5, we close the discussions with a few remarks.

2. The evidence interpretation of paraconsistency

From a pluralistic point of view, one can argue that different logics preserve different things. Taking the example of classical and intuitionistic logics, one could say that they formalize different notions of logical validity. While classical logic formalizes preservation of truth, intuitionistic logic formalizes preservation of constructive provability. In this sense, there is no rivalry between these logics, since truth and constructive provability are different things. Inspired in this conciliatory form of logical pluralism, Carnielli and Rodrigues defend that paraconsistent logics should be seen as preserving evidence. In Carnielli and Rodrigues (2019a, b, c), the authors define evidence for a proposition A as *reasons for believing or accepting A*. Under this interpretation, such reasons may be wrong, giving rise to contradictions. Given the evidence interpretation, we have the following scenarios concerning evidence:

Evidence only for A;
Evidence only for $\neg A$;
Conflicting evidence for A and $\neg A$;
No evidence for A nor for $\neg A$.

These four scenarios suggest that the logic that formalizes preservation of evidence fails to validate both *explosion rule* (Exp: $A, \neg A \models B$) and the *Principle of Excluded Middle* (PEM: $\models A \vee \neg A$). That is, the logic must be both paraconsistent and paracomplete. The system they present to formalize this informal notion is the logic BLE. This logic has the language $\mathfrak{L}$ of Classical Propositional Logic and it is presented by a natural deduction proof system:[2]

Definition 2.1. (*Carnielli & Rodrigues 2019a) The proof system for* BLE *is characterized by the following inference rules:*

Rules for conjunction:

$$\frac{A \quad B}{A \wedge B}(\wedge I) \qquad \frac{A \wedge B}{A}(\wedge E) \qquad \frac{A \wedge B}{B}(\wedge E)$$

Rules for disjunction:

$$\frac{A}{A \vee B}(\vee I) \qquad \frac{B}{A \vee B}(\vee I) \qquad \frac{A \vee B \quad \dfrac{[A]}{C} \quad \dfrac{[B]}{C}}{C}(\vee E)$$

Rules for implication:

$$\frac{\dfrac{[A]}{B}}{A \to B}(\to I) \qquad \frac{A \quad A \to B}{B}(\to E)$$

Rules for negation:

$$\frac{A}{\neg\neg A}(\neg\neg I) \qquad \frac{\neg\neg A}{A}(\neg\neg E)$$

Rules for negation and conjunction:

$$\frac{\neg A}{\neg(A \wedge B)}(\neg \wedge I) \qquad \frac{\neg B}{\neg(A \wedge B)}(\neg \wedge I) \qquad \frac{\neg(A \wedge B) \quad \dfrac{[\neg A]}{C} \quad \dfrac{[\neg B]}{C}}{C}(\neg \wedge I)$$

[2] For the basic notions of natural deduction, one can consult Sundholm (1983).

Rules for negation and disjunction:

$$\frac{\neg A \qquad \neg B}{\neg(A \vee B)}(\neg \vee I) \qquad \frac{\neg(A \vee B)}{\neg A}(\neg \vee E) \qquad \frac{\neg(A \vee B)}{\neg B}(\neg \vee E)$$

Rules for negation and implication:

$$\frac{A \qquad \neg B}{\neg(A \to B)}(\neg \to I) \qquad \frac{\neg(A \to B)}{A}(\neg \to E) \qquad \frac{\neg(A \to B)}{\neg B}(\neg \to E)$$

Inspired in the idea of Proof-Theoretic Semantics, Carnielli and Rodrigues (2019a, b) defend that the meaning of the connectives of BLE is given by their corresponding inference rules. That is, the rules of Definition 2.1 formalize the notion of preservation of evidence. So, it is necessary to show that each rule of BLE is adequate to formalize the intended informal notion, in a similar spirit that Brouwer-Heyting-Kolmogorov intepretation gives meaning to the intuitionistic connectives. For example, if k is an evidence for $A \wedge B$, then k is an evidence for A and B. The rule (∧I) is justified as follows: "if k and k_0 are evidences, respectively, for both A and B, k and k_0 together constitute evidence for $A \wedge B$" (Carnielli & Rodrigues, 2019a, p. 3796).[3] Similar arguments have to be applied to justify the other inference rules of BLE. The model-theoretical counterpart of BLE is a valuation semantics presented by the following definition.

Definition 2.2. *Let* For_{BLE} *be the set of formulas of* BLE. *A* semivaluation *b is a function from* For_{BLE} *to the set* {1, 0} *defined as follows:*

1. *if* $b(A) = 1$ *and* $b(B) = 0$, *then* $b(A \to B) = 0$;
2. *if* $b(B) = 1$, *then* $b(A \to B) = 1$;
3. $b(A) = 1$ *and* $b(B) = 1$ *iff* $b(A \wedge B) = 1$;
4. $b(A) = 1$ *or* $b(B) = 1$ *iff* $b(A \vee B) = 1$;
5. $b(A) = 1$ *iff* $b(\neg\neg A) = 1$;
6. $b(\neg(A \wedge B)) = 1$ *iff* $b(\neg A) = 1$ *or* $b(\neg B) = 1$;
7. $b(\neg(A \vee B)) = 1$ *iff* $b(\neg A) = 1$ *and* $b(\neg B) = 1$;
8. $b(\neg(A \to B)) = 1$ *iff* $b(A) = 1$ *and* $b(\neg B) = 1$;

[3] Fitting (2017) proves that BLE is translatable in the modal logic KX4, which is obtainable by S4 by substituting the axiom []A→A by [][]A→[]A. In KX4, the operator [] is intended to interpreted non-factive evidence.

A semivaluation is a valuation *if it satisfies the following condition:*

(Val) *For all formulas of the form* $A_1 \to (A_2 \to \ldots \to (A_n \to B)\ldots)$*, where* B *is not of the form* $C \to D$*: if* $b(A_1 \to (A_2 \to \ldots \to (A_n \to B)\ldots)) = 0$*, then for all* A_i, $1 \leq i \leq n$, $b(A_i) = 1$ *and* $b(B) = 0$.

Let $\Gamma \subseteq For_{\text{BLE}}$ *and* $A \in For_{\text{BLE}}$*. The relation* $\Gamma \mid= B$ *is defined as follows: if* $b(A) = 1$*, for all* $A \in \Gamma$ *then* $b(B) = 1$.

Valuation semantics are often criticized for the fact that they do not offer a satisfactory explanation of the meaning of logical constants.[4] The authors defend that the valuations of Definition 2.2 should not be seen as defining the meaning of the constants of BLE. Such valuations are taken as tools for proving metatheoretical results such as soundness and completeness. Antunes et al. (2020) provide a Kripke semantics for BLE, where the logical connectives of this logic are interpreted in a more intuitive way. But, as we said before, the natural deduction rules of Definition 2.1 are taken to give meaning to the connectives of BLE. If one maintains that classical logic is the system that best formalizes truth-preservation because it respects Exp and PEM, then BLE is not able to express preservation of truth. In order to do so, it is necessary to extend BLE with a device to recover classicality. Carnielli and Rodrigues (2019a) extend the language of BLE with the connective *o*, obtaining the *Logic of Evidence and Truth* (LET_J). As we said before, "*oA*" means "there is a conclusive evidence for *A*".This logic extends the natural deduction system for BLE with inference rules involving the connective *o*. The definition goes as follows:

Definition 2.3. *The logic* LET_J *is obtained by extending Definition 2.1 with the following rules:*

$$\frac{\circ A \qquad A \qquad \neg A}{B}(Exp^\circ) \qquad \frac{\overline{\circ A} \qquad \frac{[A]}{B} \qquad \frac{[\neg A]}{B}}{B}(PEM^\circ)$$

The rule (*Exp*°) is the *gentle principle of explostion*, which says that if we have a conclusive evidence for A and have an evidence for A

[4] We refer the reader to Carnielli (1990) for a more elaborated criticism of bivaluations.

and $\neg A$, then we have evidence for every B. The rule (PEM^{o}) expresses a more restricted version of excluded middle, the rule of *gentle excluded middle* that can also be expressed as follows:

$$oA \vDash A \vee \neg A \qquad (1)$$

That is, excluded middle holds for formulas that we have conclusive evidence. The semantic definition of LET_J extends Definition 2.2 with a clause for formulas oA.

Definition 2.4. *Let* For_{LETJ} *be the set of formulas of* LET_J. *A* semivaluation *b is a function from* For_{LETJ} *to the set* $\{1, 0\}$ *having the clauses 1-8 of Definition 2.2 with the following clause:*

9. $b(oA) = 1$ *implies* [$b(\neg A) = 1$ *iff* $b(A) = 0$].

Since LET_J is paraconsistent and validates (Exp^{o}), LET_J is a *Logic of Formal Inconsistency* (Carnielli et al., 2007). Moreover, LET_J is paracomplete and validates (PEM^{o}). So, LET_J is also a *Logic of Formal Underdeterminedness* (LFU, Marcos, 2005). Then, it is both a LFI and a LFU. LET_J is able to represent truth as follows:

$A \wedge oA$ means "A is true".
$\neg A \wedge oA$ means "A is false".

This shows that classical and paraconsistent logics can coexist without rivalry, because it is possible to talk about classical sentences by labelling them with the connective o. The following result shows that LET_J recovers classical inferences.

Theorem 2.5 (Carnielli & Rodrigues, 2019a) *Suppose that* $o\neg^{n1}A_1, \ldots, o\neg^{nm}A_m$ *hold for* $0 \leq n_i$ (*where* $0 \leq n_i$ *represents the number of* n_i *occurences of negation before the formula* A_i). *Then*:

Any LET_J - *formula with* $A_1, \ldots, A_m$ *over* $\{\neg, \wedge, \vee, \rightarrow\}$ *behaves classically*.

As Carnielli and Rodrigues (2019a) argue, Theorem 2.5 is a form of *Derivability Adjustment Theorem* (DAT). Roughly speaking, DAT states that classical inferences hold when we have conclusive evidence for the sentences involved in the inference. According to Rodrigues and

Carnielli (2022), such a recovering device highlights the anti-dialetheist perspective of their approach because "the simultaneous truth and falsity of A is expressed by $A \wedge \neg A \wedge oA$, and the latter leads to triviality" (Rodrigues & Carnielli, 2022, p. 325). So, according to them, paraconsistency does not rival classical logic. From this conciliatory point of view, the logic that formalizes the epistemic reading of the logical constants should have devices to formalize the discourse about truth.

From a philosophical perspective, Carnielli and Rodrigues' proposal is interesting because it shows that paraconsistent logics does not commit us to an ontological position that asserts the existence of true contradictions, such as *dialetheism* (Priest et al., 2022). From a more logical perspective, their formalism shows that it is possible to recover classical inferences as much as possible. With the aid of the connective o, it is possible to recover classical inferences by stipulating that there is conclusive evidence for the sentences in question. In the next Section, we present some criticisms that the epistemic approach proposed by Carnielli and Rodrigues have received in the literature.

3. Objections to the evidence interpretation of BLE and $\mathbf{LET_J}$

Carnielli and Rodrigues' evidence interpretation of paraconsistency faces some objections in the literature. Barrio (2018) argues for the distinction between *pure logics* and their *philosophical interpretations*. Pure logics are formal systems endowed with a consequence relation, whereas philosophical interpretations can be understood as an informal interpretation of the logical constants of a logical system. He argues that the same logical system may receive different philosophical interpretations, but the system itself has no intrinsic connection with these interpretations. In his paper, Barrio argues for the possibility of interpreting BLE and LET_J in alethic terms. So, oA would mean "A is not a dialetheia."[5] This shows that a dialetheist can use the recovering device in his/her logic.

We can illustrate Barrio's argument that dialetheism is compatible with recovery devices by another logic as an example. Consider the case

[5] Barrio and Da Ré (2018) also argue for the distinction between pure logics and their philosophical interpretations. As they make it clear, paraconsistency is a formal feature of some logical systems, and dialetheism is an ontological thesis about the existence of true contradictions. According to them, there is nothing in the paraconsistent formal systems that obligates one to accept dialetheism. Indeed, it is possible to interpret paraconsistent logics in non-dialetheic terms, and it is also possible to interpret non paraconsistent logics according to dialetheism.

of the *logic of paradox* LP (Asenjo, 1966; Priest, 1979) that shares the same propositional language $\mathfrak{L}$ as BLE, and is usually presented as follows:

Definition 3.1. (Asenjo, 1966; Priest, 1979) The logic LP has the matrix $M = (\{1,½,0\}, \neg, \rightarrow, \{1,½\})$ whose operations $\neg$ and $\rightarrow$ have the following truth tables:

	¬
1	0
½	½
0	1

→	1	½	1
1	1	½	0
½	1	½	½
0	1	1	1

The valuations $v\colon For_{LP} \rightarrow \{1,½,0\}$ of LP are defined as follows:
$v(\neg A) = 1 - v(A)$;
$v(A \rightarrow B) = \max(1 - v(A), v(B))$.

The set of all valuations $v\colon For_{LP} \rightarrow \{1,½,0\}$ constitutes the semantics of LP, denoted by sem_{LP}. We say that a valuation $v \in sem_{LP}$ is a *model* for the formula $A \in For_{LP}$ if $v(A) \in \{1,½\}$. Let $\Gamma \subseteq For_{LP}$ be a set of formulas. We say that v is a model of Γ if v is a model for every $A \in \Gamma$. We say that A is *consistent* if A has a model. A is a *tautology* if every $v \in sem_{LP}$ is a model of A. The relation $\vDash_{LP} \subseteq \wp(For_{LP}) \times For_{LP}$ of *logical consequence* is defined as follows: $\Gamma \vDash_{LP} A$ if and only if every model of Γ is a model of A.

Priest (1979, 2006) defends that LP characterizes the basic principles of dialetheism.[6] He interprets the truth-values of the set {1,½,0} as follows: 1 stands for truth *simpliciter*, ½ stands for true and false, and 0 stands for false *simpliciter*. Moreover, he also recognizes the need for recovering some classical inferences that are used in every-day reasoning, such as *modus ponens*, which is invalid in LP. As he recognizes, classical logic holds in consistent scenarios.[7] So, in principle, it could be possible to extend LP with the operator *o* that has the following truth-table:

[6] Lewis (1982) argues that the truth-values of LP can also be interpreted epistemologically: 1 stands for *truth in all disambiguations*, 0 stands for *false in all disambiguations*, and ½ stands for *truth in some disambiguations and false in others*.

[7] Indeed, such move has been done in Antunes (2018). But there, he considers the logic LFI1 (Carnielli et al., 2004).

	o
1	1
½	0
0	1

In this scenario, oA means that "A is consistent". So, in the presence of this connective, we can represent truth *simpliciter* and false *simpliciter* in the new logic LP^o in the same way as in the case of LET_J.[8] That being said, dialetheism is compatible with the acceptance of some classical inference rules in contexts that we know to be consistent, and this justifies a possible introduction of a consistency connective o.[9]

It is a matter of fact that a single formal logical system can receive different interpretations, because the formal definitions of consequence relations are not able to uniquely point to a single informal notion of validity, and the case BLE and LET_J are not exceptions.[10] Although Rodrigues and Carnielli (2022) accept that both BLE and LET_J can receive alternative interpretations, they claim that it is the combination of BLE with its evidence informal interpretation that is anti-dialetheist by nature. So, it is not these systems alone that are incompatible with dialetheism, but we also must consider the evidence informal interpretation. This combination is, according to them, incompatible with the ontological interpretation of paraconsistent.

Although Carnielli and Rodrigues are correct in saying that their epistemological interpretation is anti-dialetheist, their systems are compatible with a dialetheist interpretation. Of course, formal systems themselves may prevent certain informal interpretations, as in the case of intuitionistic logic to classical truth. Carnielli and Rodrigues certainly agree with the latter statement, and some textual evidence shows it (Rodrigues & Carnielli, 2022). However, as we argued, this does

[8] One can find a study of LP^o in Barrio et al. (2017).

[9] A parallel can be done in the case of classical logic versus intuitionistic logic. An intuitionistic logician that considers intuitionistic logic as the only true logic can recognize that under certain circumstances the excluded middle holds. For example, in finite domains, he/she recognizes the truth of such principle. The dispute arises in domains whose cardinality is bigger than ω. Then, the paraconsistent logician who supports LP may also accept the validity of instances of modus ponens when only truth *simpliciter* is at issue.

[10] In Bezerra and Venturi (2021), it is argued that the formal notions of validity are underdetermined with respect to their informal notions.

not happen with the logics BLE and LET_J regarding the ontological interpretation.

The second objection comes from Lo Guercio and Szmuc (2018). As we presented before, Rodrigues and Carnielli understand evidence as reasons for believing/accepting the truth. However, as Lo Guercio and Szmuc argue, the acceptance of a proposition must be understood as a rational acceptance. Thus, given two conflicting evidences for *A*, one should conclude that there is an evidence for $A \wedge \neg A$, by considering BLE's rule ($\wedge$I). In this sense, Rodrigues and Carnielli's evidence interpretation of paraconsistency requires an extremely permissive notion of acceptance, which is controversial among epistemologists. Then, given a pair of contradictory evidences of *A*, Lo Guercio and Szmuc defend that the most acceptable attitude in view of such a contradiction is to suspend the judgement about *A*.

Lo Guercio and Szmuc's objections against the evidence interpretation of paraconsistency also appear among Arenhart (2021)'s criticisms against Carnielli and Rodrigues' approach. According to Arenhart, evidence does not seem to require a paraconsistent approach.[11] First, the evidence interpretation of paraconsistent logics does not do justice to most of paraconsistent logics since most of them validate (PEM). So, Carnielli and Rodrigues' proposal of BLE and LET_J is better understood as a paraconsistent account of evidence rather than evidence interpretation for paraconsistent logics. Second, given the problems raised by Lo Guercio and Szmuc, Arenhart argues that paraconsistent adjunctive logics are not adequate approaches to evidence. As we can see, both Arenhart's and Lo Guercio and Szmuc's objections concern the validity of the rule of introduction of conjunction. That is, the evidence interpretation of the logical constants requires a non-adjunctive logic.

According to Carnielli and Rodrigues (2019a, c), if we have two documents where one is evidence for *A* and the other is evidence for $\neg A$, a new document that puts together these two documents is evidence for $A \wedge \neg A$. They argue that these situations are possible when evidence is not conclusive. Of course, when we have conclusive evidence for a statement *A*, these situations do not occur. However, although this example is an interesting one, we do not think it is enough to justify contradictory evidence from a more general perspective. It seems to us that this justification for adjunction commits the fallacy of the

[11] This objection also appears in Arenhart (2022). Arenhart and Melo (2022) argue that the evidence approach to paraconsistency struggles when dealing to semantic paradoxes, such as the liar paradox. We refer the reader to both papers to consult a more detailed criticism.

existential quantifier. If a person looks at a pyramid *f* from two different perspectives, she can have contradictory evidences about the geometrical form of *f*. Looking from the perspective *p1*, she has evidence that *f* is a triangle. From the perspective *p2*, she has evidence that *f* is a square. But from no perspective, she will have evidence, reasons for believing, that *f* is a triangle and a square.[12]

It seems to us that the source of the problem lies in the existential requirement of the interpretation of rule (∧I). For the moment, let e_1:*A* and e_2:*B* denote that e_1 (respectively, e_2) is an evidence for *A* (respectively, for *B*). By the informal explanation given by Carnielli and Rodrigues to justify the validity of (∧I), we have that: if there is an evidence e_1 for *A* and there is an evidence e_2 for *B*, then e_1 and e_2 are evidences for *A*∧*B*. But how to put together these two evidences in order to create a new evidence e_3? If we were dealing with a strong and factive kind of evidence such as constructive provability, the problem would become clearer because proofs are monotonic. In the case we are dealing with consistent theories, we do not have factive evidences for *A* and ¬*A* in the same theory. So, in this case, it is possible to create a proof of a conjunction given a proof of the conjuncts. But, as Lo Guercio and Szmuc's and Arenhart's objections and the above examples point, it is not immediate at all how to obtain in general a possibly non-factive evidence for a conjunction from non-factive evidence of the conjuncts.

Let us consider again Arenhart's first objection to Carnielli and Rodrigues' proposal. As he points out, the evidence interpretation of paraconsistency does not do justice to most of paraconsistent systems because the vast majority of these logics validate (PEM).[13] In this sense,

[12] Arenhart (2021, p. 11552) presents counterexamples from quantum mechanics: "In a double slit experiment, there is evidence that an item is a particle (when only one slit is open), and also, there is evidence that the item is a wave (when both slits are open). So, according to the behavior of evidence as described by BLE, there is evidence that the item is a particle and a wave. But there is no such evidence in quantum mechanics. There is no way of putting the evidences together in this case."

[13] One of the referees criticized this objection against Carnielli and Rodrigues's proposal in Carnielli and Rodrigues (2019a). As she/he pointed out, Carnielli and Rodrigues presented a paraconsistent and paracomplete system that formalizes the notion of evidence, and this does not mean that their proposal comprehends all paraconsistent systems. On the other hand, we can find textual evidence where the authors claim that evidence is a "well-suited to a non-dialetheist reading of paraconsistency" (Carnielli et al., 2021, p. 5462). If we understand paraconsistency as a field of study (*e.g.,* da Costa et al., 2007; Priest et al., 2022), including a wide range of logics, the former quote can be faced in a more general perspective. According to Arenhart (2021), Carnielli and Rodrigues' proposal entangles two things: a*pproaching paraconsistency with evidence*, in the sense that evidence confers meaning to

Carnielli and Rodrigues' should be better seen as a paraconsistent approach to evidence. But, as some textual passages testify, they defend that evidence should be taken as the informal notion underlying paraconsistent systems.[14] Now, since they defend that evidence is an informal notion better formalized by a logic that is both paraconsistent and paracomplete, then we do not know if evidence tends more to paraconsistency than to paracompleteness. This causes a problem for the proponents of both BLE and LET_J. If evidence requires the failure of (Exp) and (PEM), what reasons do we have to say that evidence is a well-suited informal notion behind paraconsistency? It seems that paraconsistency and paracompleteness are on equal level in this equation because both are required to formalize evidence. Carnielli and Rodrigues cannot say that evidence is more paraconsistent than paracomplete. This would require a further step that is not provided by both authors yet.

If we do not have an ultimate reason to defend that evidence is indeed a genuine paraconsistent notion, we can equally say that paracomplete systems must be interpreted in terms of evidence. Indeed, depending on the way that we understand evidence, we have more reasons to defend that evidence is better formalized by a paracomplete system that is not paraconsistent. Intuitionistic logic is a clear example of this because evidence may be interpreted as availability of a proof. In this sense, evidence itself does not seem to be a good candidate for paraconsistent logics, because it requires too much from those willing to adopt a paraconsistent logic. In this sense, despite being philosophically controversial, the dialetheist approach to paraconsistency is more inclusive than the evidence interpretation of paraconsistency concerning the choice of logical systems. The reason is simple: the dialetheist interpretation of paraconsistency does not necessarily require the failure of (PEM). The other logical constants are up for dispute.[15] That is, a dialetheist is not forced prima facie to accept truth-value gaps, but only truth-value gluts.

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paraconsistency, versus *approaching evidence with a paraconsistent system*, in the sense that paraconsistent as a formal tool to formalize reasoning with evidence. *As he argues, approaching paraconsistency with evidence does not do justice to most of paraconsistent logical systems, because most of them validate (PEM).*

[14] See footnote 13.

[15] The case of LP is an interesting one. LP does not validate *modus ponens* (A, $A \to B \models B$). On the other hand, Priest (2006) argues for the need to extend LP with a relevant conditional that validates *modus ponens* if one wants to have paraconsistent logic as a basis of robust semantic theories. This shows that dialetheism is compatible with stronger logics than LP.

4. On the interpretation of *o* in LET_J

The connective *o* has fundamental importance in the epistemological interpretation of paraconsistency proposed by Carnielli and Rodrigues. Indeed, the idea of isolating the inconsistent statements from the consistent ones (i.e., non-contradictory) by tagging the latter statements with *o* plays a fundamental role in the Brazilian approach to paraconsistency. In the formulation of the hierarchy of paraconsistent systems C_n ($1 \leq n \leq \omega$), da Costa (1974) requires that they should contain most part of classical logic. His motivations are clear: his paraconsistent systems do not rival classical logic. Instead, they are intended to be complementary tools for the formal study of paradoxes and contradictions. In his terminology, "A^o" means "*A* is non-contradictory".[16] Now, we can see that the introduction of the connective *o* in the language of LET_J perfectly matches well with this idea.

However, there is a problem concerning some principles of *o* that LET_J does not validate. This will pose a problem in the relation between LET_J and its intended interpretation. In general, when one proposes a logical system *L* to describe some informal notion *I*, it is expected that *L* sets general principles that *I* validates. That is, *L* is expected to formalize the general valid intuitions about *I*. On the other hand, when *L* fails to formalize *I*, *L* may fail in two ways: (1) the axioms and rules of *L* are not *faithful* with respect to *I* in the sense that the theorems of *L* are not valid with respect to *I*; and (2) *L* may not be *adequate* with respect to *I*, in the sense that the formalizations of *I* in the language of *L* are not valid in *L*.[17]

From Arenhart's and Lo Guercio and Szmuc's objections to BLE (a fortiori to LET_J), we infer that LET_J is not faithful to the evidence interpretation due to the introduction rule of conjunction. A possible way to deal with this problem would be to restrict the application of (∧I) to statements that have conclusive evidence. That is,

[16] According to Carnielli et al. (2020), da Costa makes two important contributions to paraconsistency. The first contribution was the introduction of the connective o as an expressive resource to separate non-contradictory statements from contradictory ones. The second contribution was the idea that intuitionistic logic and the first system of the hierarchy Cn, C1, are dual because $\neg(A\wedge\neg A)$ and $A\rightarrow\neg\neg A$ fail in C1 but not in intuitionistic logic, and (PEM) and $\neg\neg A\rightarrow A$ fail in intuitionistic logic but not in C1. However, as Carnielli et al. argue, da Costa mistakenly identifies the failure of the $\neg(A\wedge\neg A)$ and $A\rightarrow\neg\neg A$ as the source of the duality between intuitionistic and paraconsistent logics. They argue that the duality between these logics lies in the failure of (Exp) on paraconsistent logics and the failure of (PEM) in intuitionistic logic.

[17] The notion of faithfulness and adequacy are taken from Shapiro (2005).

$$\frac{\circ A \quad \circ B \quad A \quad B}{A \wedge B}(\wedge^{\circ} I)$$

Such a rule should respond to the objections mentioned above, since we do not have conclusive evidence for contradictory pairs of formulas, then rule ($\wedge^o$I) should not give rise to the problems pointed out by Arenhart and Lo Guercio and Szmuc. Since the LET_J is monotonic, one could easily derive ($\wedge^o$I) from ($\wedge$I). The converse is obviously not the case. Thus, if we take the rule ($\wedge^o$I) as primitive instead of ($\wedge$I), we obtain a fragment of LET_J, which will not be investigated here. Given this scenario, we have that LET_J is not faithful with respect to the evidence interpretation. Now we will argue that LET_J is not adequate either. That is, the notion of evidence validates some principles, especially about conclusive evidence, whose formalizations are not valid in LET_J.

As Definition 2.3 shows, LET_J has neither introduction nor elimination rules for *o*. According to Carnielli and Rodrigues (2019b), formulas of the form *oA* must be introduced from outside the system. The system itself does not decide what formulas have conclusive evidence and what formulas do not. Here we will focus on propagation principles of *o*. Consider the following formula:

$$o(A \wedge B) \rightarrow (oA \wedge oB) \quad (2)$$

It is easy to see why formulas like (2) are not valid: if, for example, A = ⊥ then we have *oA* and $o(A \wedge B)$ no matter the value of *B*. If we have contradictory evidences for *B*, then it is not the case that *oB*, and so $oA \wedge oB$ does not hold. It is reasonable to maintain that formulas such as (2) do not hold. On the other hand, some statements are intuitively valid about evidence, and LET_J does not validate them. For example:

$$(oA \wedge oB) \rightarrow o(A \wedge B) \quad (3)$$
$$(oA \wedge oB) \rightarrow o(A \vee B) \quad (4)$$

Indeed, as Antunes et al. (2020) emphasize, we do not obtain the consequent $o(A \wedge B)$ in (3) because the semantic clause of *o* does not give us sufficient semantic information about *A* and *B*, and a similar reason applies to (4). However, in what follows, we argue that both formulas should be valid according to the evidence interpretation of LET_J. In the case of (3), the argument goes as follows: suppose that there is conclusive evidence for *A* and conclusive evidence for *B.* According to Carnielli and

Rodrigues, there is conclusive reason for accepting A and conclusive reason for accepting B. Given that conjunction behaves as in classical logic and both A and B have classical behavior, we can conclude that there is conclusive evidence for $A \wedge B$. In the case of (4), the argument holds for a similar reason.

Antunes et al. (2020) could respond to our criticism by pointing that LET_J validates some form of propagation over the propositional connectives, since Theorem 2.5 holds. As they observe, Theorem 2.5 does not mean that oA and oB imply $o(A \wedge B)$ (respectively, $o(A \vee B)$). However, even if this theorem allows LET_J to recover classical inferences with the aid of the connective o, this logic should allow us to make inferences when we have formulas of the form $o(A * B)$, for $* \in \{\wedge, \vee, \rightarrow\}$. As we will argue below, this is a relevant point because logics should be normative about the informal notions they formalize.

A similar phenomenon happens with modal logics. Even if the logic K has general principles governing the modal operator [], it should be extended with more specific modal axioms in order to formalize a desired notion such as necessity, knowability, provability, obligation, and so on. The generality of the logic K suggests that it is too vague to capture a specific notion. In the case of LET_J, we can argue that the inference rules of LET_J are too broad to draw conclusions about principles involving conclusive evidence.[18] In fact, it is possible to extend LET_J with more specific principles concerning o. But, as presented before, one should also take into consideration that the faithfulness of ($\wedge$I) is still under dispute.

It is important now to situate Carnielli and Rodrigues' proposal within the debate about logical pluralism. In their works about the philosophical interpretation of paraconsistent logics, they seem to support a kind of *contextualist pluralism*. Such a version of pluralism is not new in the literature, and it was recently defended by Caret (2017).[19] Roughly speaking, Caret defines contextualist pluralism as a position that holds that different logics L capture different *deductive standards*. Caret understands deductive standard as an "admissible class of cases that function as logically salient alternatives" (Caret, 2017, p. 753). In this sense, different informal notions of validity characterize different

[18] This claim is a modification of Ferguson (2018)'s argument against the interpretation of o as consistency in the weakest logic of the family of LFIs, the logic mbC.

[19] Interestingly, da Costa's pluralistic motivations behind the introduction of his hierarchy of paraconsistent systems constitute a version of contextualist pluralism, even if he calls his position relativistic in his book (da Costa, 1980).

deductive standards.[20] According to this version of pluralism, the meaning of "valid" is local. Logical validity is, in a certain sense, relative to the property one wants to preserve in the arguments.

In Rodrigues and Carnielli (2022, p. 329), the authors defend that a unique informal notion of validity "should not be compatible with two different formal systems." For example, classical logic and LP do not capture the same notion of truth, because classical logic is stronger than LP. The same reasoning applies to any pair of logics proposed to capture a given informal notion I. So, considering that each informal notion characterizes a context, we obtain that different logics capture different informal notions. It is not the case that a single informal notion is captured by two different logics. Then, while classical logic deals with preservation of truth, LET_J intends to deal with preservation of evidence. Under this kind of pluralism, we can understand the normativity of logic in a more restricted form: if classical logic is normative with respect to preservation of truth, then it is to be expected that this logic sets the general principles that govern this informal notion. The same applies to intuitionistic logic concerning the informal notion of constructibility.[21]

Under such restricted understanding of normativity, LET_J then should be taken as normative in the context of preservation of evidence. However, by the fact that LET_J does not validate intuitive principles about conclusive evidence, and by the criticisms raised in Section 3 concerning the faithfulness of (∧I), LET_J is not normative even in its more restricted understanding. That is, LET_J is not normative in the context of preservation of evidence. To make our point more precise, consider as a title of comparison the logic LFI1 (Carnielli et al., 2004), which was proposed to be a formal system capable to handle information in evolutionary databases. This paraconsistent logic was proposed to capture preservation of information in complete scenarios. Because LFI1 is a LFI, it contains a connective o that tags propositions carrying reliable information. In this logic, it is also up to the user to ascribe which propositions have reliable information and which do not. But, different from LET_J, LFI1 has sufficient deductive power to tell us what to do when we have formulas preceded by o. So, considering that the logical constants of LFI1 formalize preservation of (possibly inconsistent) information and that o captures a notion of reliable information, we are allowed to say that LFI1 is normative with respect to both reliable and

[20] We refer the reader to Caret (2017) for more details.

[21] If classical logic, for example, has more than one informal interpretation, this logic sets general principles for these informal notions.

non-reliable information, when we consider complete scenarios. Now, since LET_J is not able to tell us what to do with respect to formulas of the form $\circ A$, and does not validate formulas like (3) and (4), we are entitled to say that LET_J is not normative with to preservation of conclusive evidence.

It is important for the moment to make it explicit what is the underlying notion of information we are dealing with. Here we are considering Dunn's conception of information (Dunn, 2008). In Dunn's words, information is "what is left from knowledge when you subtract, justification, truth, and belief" (Dunn, 2008, p. 581). Belnap (1977) presents the system FDE, which is argued to capture this notion of information in databases.[22] In the case of LFI1, such a notion of information works if we presume complete scenarios because LFI1 validates (PEM). If we want to consider other aspects of the notion of information, such as the notion of *actual information* (i.e., the information that is practically available to a given agent), maybe paraconsistent logics will not be adequate systems. For example, D'Agostino (2015) argues that the logic formalizing such a notion of information should be non-deterministic. For him, the basic notions of the formal semantics that regulate actual information are *informational truth* (holding information that a sentence A is true) and *informational falsity* (holding information that a sentence A is false). In this case, (PEM) will not hold because an agent may have no information at all about a given sentence A. On the other hand, (Exp) holds because "no agent can actually possess both the information that A is true and the information that A is false would be deemed to be equivalent to possessing no definite information about A" (D'Agostino, 2015, pp. 83-84).[23]

The objections raised in Sections 3 and 4 do not imply that LET_J cannot be interpreted epistemologically. They only show that the evidence interpretation of LET_J is problematic, but nothing prevents that this logic can be interpreted epistemologically in terms of other notions. According to Wansing (2022), logic can be seen as the study of

[22] Blasio (2017) provides a nice epistemological interpretation for FDE, where each truth-value of FDE may be interpreted in terms of propositional attitudes of acceptance, rejection, non-acceptance and non-rejection.

[23] The non-deterministic character of D'Agostino's proposal is justified in the following passage: "In general, when we are faced with a conjunction $A \wedge B$ in which A and B are indeterminate, the value of the conjunction may be either informational falsity 0, or informational indeterminacy $\bot$, depending on whether or not we hold the additional information that A and B cannot be simultaneously true." (D'Agostino, 2015, p. 85). We refer the reader to his paper for further details.

the informational flow. He argues that this informational understanding leads to the semantic values T, B, N, and F of the logic FDE that are understood as follows: T means "told only true," F means "told only false," B means "both told true and told false," and N means "neither told true nor told false." As it is known, the logic N4 (Almukdad & Nelson, 1984) extends FDE with a stronger conditional connective. Thus, it is reasonable to argue that N4 is compatible with this informational reading. Since BLE is equivalent to N4 (Carnielli & Rodrigues, 2019a, c), then BLE is also compatible with such an interpretation (Antunes et al., 2020). Now, in the case of LET_J, the connective *o* will serve to isolate inconsistent information from consistent ones. On the other hand, we still maintain that LET_J should be extended in a way to make it possible to analyse formulas of the form $o(A{*}B)$, for $* \in \{\wedge, \vee, \rightarrow\}$. This means that LET_J should be extended with rules that regulate the interaction of the binary connectives with *o*. This will be investigated in a further work.[24]

5. Conclusion

The relation between logical systems and their informal interpretation gives rise to one of the most fruitful debates in the philosophy of logic. The arguments raised in this paper concerning the epistemic interpretation of paraconsistency show how difficult it is to interpret the logical constants of a particular logical system. Even if these formal systems are underdetermined with respect to their informal interpretations, it is quite natural any attempt to interpret them. If a system L is not amenable to an informal interpretation that gives meaning to the logical constants of L and to its consequence relation, then one may suspect about its status as a system of logic.[25] That is, given that logics are expected to be normative for the informal notion that they formalize, one could question the status of logic a formal system devoid of any informal interpretation/application. Of course, this is not to say that formal calculi are unimportant. Indeed, many formal calculi are worth studying to investigate metatheoretical properties from a general perspective.[26] Moreover, an uninterpreted formal system

[24] It is important to say that we are not defending that this is the only application of BLE and LET_J. Both logics are interesting tools for analysis of non-trivial inconsistent theories. In this paper, we analyzed the evidence interpretation of both logics, but nothing prevents other informal intepretations for these logics.

[25] Rescher (1969), for example, argues that formal systems devoid of interpretations cannot be called a system of logic.

[26] As one of the referees pointed out, our position about formal systems without

can become useful to formalize an informal notion of validity, and this is indeed an interesting philosophical attitude towards formal calculi.[27]

As we argued, BLE and LET_J are neither faithful nor adequate with respect to its informal interpretation of evidence. We also saw that they may be interpreted in terms of a rather informal notion of information. An important improvement of this informational interpretation of these logics would be first to strengthen the logic LET_J with rules that govern the interaction of connective *o* with the other connectives of the language of this logic. Second, since the notion of information at issue is captured by more than one system (LET_J, FDE), it would be interesting to investigate what subtleties of the concept of information that is captured by LET_J and not by FDE. From a contextualist perspective of logical pluralism, it is reasonable to maintain that each deductive standard is better captured by one logic. Then, given such a clarification, the epistemological understanding of paraconsistency in terms of preservation of information will be settled.

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any informal interpretations differs from Barrio (2018)'s position about pure logics. Indeed, he does not defend that pure logics without philosophical interpretations are not logics, strictly speaking. On the other hand, we maintain that his main position that logics can receive multiple interpretations remains intact.

[27] As Rodrigues and Carnielli (2022) observe, many paraconsistent systems presented by da Costa (1974) do not have intuitive interpretations. But these systems were important to the development of the LFIs, and the latter family of logics was important to the logic LET_J.

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